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Dynamic Consequences of Stabilization Policies Based on a Return To a Gold Standard

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Abstract

Prior to the 1930's rapid inflations often occurred in conjunction with wartime finance, and were commonly halted by imposing or re-imposing a gold standard. In a general equilibrium framework I study the consequences of different methods of gold resumption for the dynamic path of the economy, including the behavior of the price level and the welfare of economic agents. In particular, I examine the implications of varying (a) the length of time between the end of the war and the return to gold; (b) the par at which gold convertibility is re-established; and (c) whether or not the government accumulates gold reserves over an extended period of time before the stabilization.

I show that a gradual post-war deflation, which permits resumption of gold convertibility as scheduled, can occur even if the money supply is not contracted and the government does not actively accumulate gold reserves. Furthermore, different policy choices regarding the resumption plan affect not only the speed of postwar deflation, but also the speed of the wartime inflation. In terms of welfare, I find that wartime generations unanimously prefer rapid resumption at high parity, while among postwar generations borrowers and lenders often have opposing interests, but on average they tend to benefit from delaying resumption and devaluing the currency.

1. Introduction

How to stabilize episodes of high inflation is a recurrent problem of monetary policy. Through much of modern history, such episodes have often occurred in conjunction with wartime finance. Under the gold standards and gold exchange standards prevalent until the 1930s, countries typically left gold during periods of wartime associated with high government expenditures in order to resort to inflationary finance. At the end of the war, or possibly some time later, the gold standard would be restored, and thereby a stabilization would be achieved. This or related patterns were observed in the United States during and after the Revolutionary War and the Civil War, in Great Britain during and after the Napoleonic Wars and World War I, and in France during and after World War I.

The repeated program of wartime suspension and postwar resumption of gold convertibility generated heated political and academic debates. Among the questions posed were the following:

(a) What determines the path of wartime inflation and the postwar price level given a credible commitment to return to gold?

(b) Given a commitment to resume convertibility at a given par of exchange, is it important to resume promptly, or is it desirable to delay resumption?

(c) Given a credible commitment to gold resumption, is it desirable to adopt a par of exchange that avoids a large scale postwar deflation?

(d) Mechanically speaking, what is required for a successful return to gold?

The determinants of the price level path during the suspension of gold payments were the focus of Madison (1779), whose arguments were commented on by Smith (1998). The Bullionist Controversy during the Napoleonic Wars concerned a similar set of issues. The desirability of rapid resumption was argued by Madison (1779) and Mitchell (1903), both of whom viewed delayed resumption as a force contributing to wartime inflation. Keynes, in his Essays on Persuasion (1931), advocated returning to gold at a par of exchange that avoids a large scale postwar deflation. The necessary preconditions of gold resumption were discussed at length by Friedman and Schwartz (1963) in the context of the American Civil War.

While these questions derive from economic history, they are also highly relevant to modern circumstances. For instance, countries desiring to stabilize moderate or high inflations today are often advised to fix their exchange rate against a stable currency. This policy, accompanied by the accumulation of foreign exchange reserves, has much in common with the adoption or re-adoption of a gold standard. The policy of using the exchange rate as a "nominal anchor" has been widely employed in many of the recent Latin American stabilizations, in the

Israeli stabilization of the mid-1980s, and in the Hong Kong stabilization plan of 1983. Thus, all these questions are deserving of modern attention.

Historically, when countries left gold — and thereby signaled that inflation was about to increase — they also made immediate promises to return to gold later. Typically these promises stipulated that the postwar resumption of convertibility would occur at the prewar par of exchange, although the timing of resumption would often be less clearly specified.[1] In this respect, too, historical precedent is relevant to modern experience. Key aspects of modern stabilization plans are also often announced well in advance, and certainly economic agents form expectations about them. The anticipation of future policy actions creates the possibility that a stabilization plan will have an impact not only on events that follow, but also on events that precede its implementation.

Following Madison (1779) and Mitchell (1903), I focus on the possibility that a credible promise of future stabilization — in particular, resumption of gold convertibility — has strong implications for the wartime path of inflation, the magnitude of postwar deflation and the welfare of various agents. In order to capture important aspects of historical experience, I consider an economy that encounters three distinct "regimes": (1) a wartime period during which a government budget deficit is financed using seigniorage revenue, (2) a post-war period when the government's budget is already balanced, but gold convertibility has not yet been resumed and a fiat currency still circulates, and (3) a post-stabilization period after the return to gold.

Under the assumption that the nature of the three regimes is known in advance, the requirements of equilibrium place joint restrictions on fiscal policy, the timing of resumption, and the postwar par of exchange. If the government accumulates gold reserves in anticipation of resumption, a rapid return to gold implies a relatively high tax burden on agents alive during the postwar, pre-resumption regime compared to what a slow return to gold would necessitate.[2] In contrast, if gold reserves are not actively accumulated in advance, and instead the entire tax burden is born by the transitional generation that experiences the reestablishment of the gold standard, rapid resumption generally decreases the tax burden necessary to achieve a given target gold parity.

Perhaps not surprisingly, my analysis indicates that there is no policy regarding the speed of resumption, the accumulation of gold reserves, and a postwar par of exchange that is

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[1] However, after the war, the date of resumption would often be announced well in advance of its actual implementation. This was the case in Britain after the Napoleonic Wars and in the U.S. after the Civil War.

[2] This burden arises as the government runs a surplus in order to accumulate gold (or foreign exchange) reserves.

uniformly preferred by all members of all generations. This is due to the fact that, as noted by Keynes (1940), policies that influence wartime inflations will typically have strong distributive consequences. Stating the results loosely, all wartime generations prefer speedy resumption at a high par of exchange, the policy advocated by Madison (1779) and, in a way, by Mitchell (1903). However, postwar generations, who are paying for the gold resumption, are typically interested in delaying resumption and lowering the par of exchange. Moreover, while the interests of creditors and debtors are well-aligned during the period of wartime inflation, they are more likely to be divergent once the war ends and the government budget is balanced or is in surplus. The differences between borrowers and lenders regarding the desired parity are particularly large in the generation that is actually responsible for executing the resumption, and if the resumption is scheduled to take place relatively early. With delayed resumption, consensus is easier to achieve. These observations help explain why there is often wartime unanimity on the desirability of rapid postwar resumption of gold convertibility, and why this unanimity evaporates once the war ends.

In addition to shedding light on the welfare consequences of various policies, my analysis also shows how the qualitative features of observed episodes of wartime finance can emerge. The typical historical pattern of economies which temporarily went off and later returned to gold is a relatively high wartime inflation, followed by a substantial postwar deflation prior to the eventual stabilization of the price level. As an example, the time path of the price level in the U.S. from 1862 to 1879 is illustrated in Figure 1. Under appropriate stabilization programs, usually associated with large postwar budget surpluses, this chain of events is replicated by the model. However, under alternative policies, which involve low postwar surpluses, the dynamics of the inflation rate can be radically different. Under these policies not only can there be a continued postwar inflation, but also a highly explosive wartime inflation.

The model predicts that one of the most important factors determining the entire price level path is the nature of the stabilization program itself. In particular, as argued by Madison (1779), even the wartime price level path may be largely under the influence of expectations regarding the future stabilization. This observation supports the proposition of the Bank of England directors during the Bullionist Controversy too, who argued that Bank of England actions had only a minor impact on the course of wartime inflation.[3] In addition, the common occurrence of a pronounced postwar deflation should be viewed as resulting primarily from the anticipation of an impending stabilization. The importance of expectations regarding future stabilization is further underlined by the fact that these expectations themselves are often sufficient to generate deflation and allow resumption to take place as scheduled, even if gold

[3] See Fetter (1965).

reserves are not actively accumulated in advance by the government. This interpretation challenges Friedman and Schwartz's account of the deflation that followed the Civil War, which attributes the deflation to exogenous factors and "makes the successful achievement of resumption on the date specified in the Resumption Act appear to be pure coincidence"[4].

My vehicle for examining these issues is a two-period lived overlapping generations model with creditors and debtors. This structure allows one to study how alternative policies affect different generations, and agents with different asset positions within the same generation. Therefore, it permits one to examine an issue that was the focus of Keynes' (1940) proposal regarding World War II financing schemes: the redistributive consequences of policies that increase or reduce the rate of wartime inflation and that also influence the behavior of the postwar price level path.

The remainder of the paper proceeds as follows. The economy considered is laid out in Section 2. Under the assumption that postwar accumulation of gold reserves is undertaken, Section 3 establishes the existence of an equilibrium and discusses the relationship between the elements of a successful stabilization program. Section 4 examines the consequences of alternative stabilization programs for the dynamic equilibrium of the economy and for the welfare of various groups of agents using numerical simulations of the model. Since some of the findings reported in Sections 3 and 4 are sensitive to the distribution of the tax burden between different types of agents, Section 5 examines the consequences of some alternative schemes for financing resumption. Finally, in Section 6 I eliminate the government's reserve accumulation program and ask how the equilibrium of the economy is affected by this change in policy. Section 7 concludes.

2. The model

The economy evolves through three consecutive regimes. Regime I is a wartime period of potentially large government budget deficits, during which the economy is off gold and the government employs inflationary finance. Regime D is a postwar period where the government budget is balanced or is in surplus, but where gold convertibility has not yet been restored. Regime S is a post-stabilization regime where a gold standard is (back) in place.

Let t=1, 2, ..., T_1, ..., T, ... index time. The periods t=1, ..., T_1 constitute Regime I, while Regime D lasts for t=T_1+1, ..., T-1. At t=T Regime S is put in place and it is preserved forever after. I assume throughout that the dates T_1 and T are known in advance.[5] I will also

[4] Friedman and Schwartz (1963), p.85.

[5] This substantially simplifies the determination of an equilibrium, and follows several of the examples in Lucas and Stokey (1983).

typically assume that $T>T_1+1$, so that there is a delay between the end of the war and the retirement of the fiat currency.

Before proceeding to the description of the specific monetary and fiscal policy arrangements under each of the three regimes, I first describe the economic environment common throughout all time periods.

THE ECONOMIC ENVIRONMENT

At each date a new generation of two-period lived agents is born. Generations are identical in size and composition[6], and consist of agents of two types. A fraction μ of agents in each generation are "borrowers", having an endowment vector of $(0, y)$, with $y>0$, whereas a fraction $(1-\mu)$ are "lenders" having an endowment vector of (v, w), with $v>0$, and $w \geq 0$. All agents receive utility from the consumption of a single, perishable good according to the utility function

$$(1) \quad u(c_1, c_2) = c_1^{\frac{1}{1+\beta}} \cdot c_2^{\frac{\beta}{1+\beta}},$$

where c_1 and c_2 denote consumption during youth and old age.

These assumptions imply that credit markets will be active at all dates. I also assume that lending is subject to a reserve requirement specifying that cash reserves must equal at least a fraction $1-\phi$ of each lender's portfolio, with $\phi \in (0,1)$.[7] Depending on the regime in place, either fiat money or gold will serve as reserves.

In addition to making loans and holding reserves, agents can also store raw gold, which depreciates at the rate of δ per period, with $\delta \in (0,1)$. I assume that there is a commonly available, *reversible* linear technology for converting consumption goods into gold and vice versa at a rate of ε units of time t good per ounce of gold.[8] The availability of this technology can have the interpretation that the economy is a small open economy within a world on a gold standard, and that the consumption good can be bought and sold abroad for gold at a constant world price and at zero transport cost.

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[6] The assumption of no population growth is inessential to the analysis.

[7] The value of ϕ is set exogenously by the government at $t=1$, and it then remains constant thereafter. It would perhaps be more realistic to assume that ϕ is reduced either at T_1, or at T, or at both dates. Allowing ϕ to vary over time this way would only strengthen my results.

[8] This structure follows Sargent and Smith (1997), and Smith (1998). Sargent and Wallace (1983) consider a piecewise linear technology for gold production. Complete reversibility constitutes a special case of their analysis.

GOVERNMENT POLICY

Monetary and fiscal policy during the three consecutive regimes is conducted as follows:

Regime I (Inflation): $t=1, \ldots, T_1$

During the wartime, the government confronts an exogenously given real per capita budget deficit of γ per period. In keeping with some of the relevant historical events[9], this temporary deficit is entirely monetized. The government's budget constraint under this regime can be written as:

$$\frac{M_t - M_{t-1}}{p_t} = \gamma, \qquad t=1, \ldots, T_1$$

where M_t and p_t are the per capita money supply and the price level at time t. The stock of fiat currency outstanding at the beginning of period 1, $M_0 \geq 0$, is given as an initial condition. This currency is held by the initial old generation.

Regime D (Deflation): $t=T_1+1, \ldots, T-1$

After the end of the war the government has no direct expenditures, but it also does not immediately redeem the outstanding stock of fiat currency. Instead, the money supply is held constant at its end-of-war level:

$$M_t = M_{T_1} \qquad t = T_1+1, \ldots, T-1.$$

During this period the government may (but need not) accumulate gold reserves in anticipation of returning to a gold standard at $t=T$. As a baseline case, I assume that gold reserves *are* accumulated by levying lump-sum taxes in periods $T_1+1, \ldots, T$, on the old agents of generations $T_1, \ldots, T-1$. Taxes collected at each date are constant over time during $t= T_1+1, \ldots, T$, and their distribution within a generation of borrowers and lenders will be discussed later. In Section 6 I will compare this scenario with an alternative where the government does not accumulate gold reserves prior to date T. In the latter case, the gold needed for resumption is acquired through taxes levied at T, and the government is essentially inactive from T_1+1 through $T-1$.

Regime S (Stabilization): $t=T, \ldots$

At date T the economy returns to a gold standard, which it retains forever after. Under such a standard, the currency consists of gold coins. In advance, the government specifies a mint ratio stipulating that each coin contains b ounces of gold per unit, and at this rate it

[9] Particularly the American Revolution.

stands ready to coin freely any gold brought to the mint. At date T the government also makes good on its promise to redeem the entire outstanding stock of fiat currency by exchanging e units of the new currency for each unit of the old currency. The value of e is also announced at t=1.

I assume that the government has no direct expenditures for $t \geq T$. However, if a_t denotes the real (goods) value of government gold reserves at t, then the time T government budget constraint requires that[10]

(2) $e \cdot M_{T-1}/p_T = a_T$.

When the government accumulates gold reserves in anticipation of restoring gold convertibility, it must levy taxes in order to do so. Let θ denote the constant value of real revenue collected during each period $t=T_1+1, \ldots, T$. Under this scenario, the evolution of the real value of the government's gold reserves, a_t, is described by

(3) $a_{t+1}=(1-\delta)a_t + \theta, \qquad t = T_1,\ldots,T-1$

Since the government accumulates no gold reserves while running a budget deficit, I set $a_{T_1}=0$. Then

(4) $a_T = \Delta \cdot \theta$,

where

$$\Delta \equiv \frac{1-(1-\delta)^{T-T_1}}{\delta}.$$

I want to be able to compare situations where the government initiates the accumulation of gold reserves prior to date T with situations where it does not. If no accumulation of gold reserves has been undertaken prior to date T, then the necessary resources for redemption are raised entirely by taxing the old members of generation T-1. In this case total taxes are denoted by τ and the real value of government gold holdings available for the redemption simply becomes

(5) $a_T = \tau$.

The remainder of this section focuses on the baseline scenario of active gold reserve accumulation. Section 6 will examine how the conclusions are affected if no gold reserves are accumulated prior to the establishment of a gold standard.

[10] Note that the price level at $t \geq T$ is expressed in units of the new currency.

INDIVIDUAL BEHAVIOR

Old agents of generations $T_1, \ldots, T\text{-}1$ are required to pay taxes during periods $t=T_1+1, \ldots, T$. In particular, each lender pays a fraction η^L, and each borrower pays a fraction η^B of their respective old age endowments to the government. The post tax old age endowments of agents in these generations are denoted by $\hat{w}$ and $\hat{y}$. In general, the post tax old age endowments of lenders and borrowers born at t, denoted by w_{t+1} and y_{t+1}, can be summarized as follows:

$$w_{t+1} = \begin{cases} \hat{w} = (1-\eta^L)\cdot w & \text{for } t = T_1, \ldots, T-1 \\ w & \text{otherwise} \end{cases}$$

and

$$y_{t+1} = \begin{cases} \hat{y} = (1-\eta^B)\cdot y & \text{for } t = T_1, \ldots, T-1 \\ y & \text{otherwise} \end{cases}.$$

The real value of total taxes collected during each period $t=T_1+1, \ldots, T$ satisfies

$$\theta = (1\text{-}\mu)\cdot\eta^L\cdot w + \mu\cdot\eta^B y.$$

Lenders

At each date there are three assets available to lenders: raw gold, consumption loans, and currency. Previous assumptions imply that the gross real return on storage of raw gold is $(1\text{-}\delta)$. Let R_t denote the gross real interest rate on consumption loans between t and $t+1$, and let Q_t denote the gross real return on real balances between t and $t+1$. Since all currency consists of fiat money during Regimes I and D, and gold coin during Regime S, the rate of return on real balances depends on the regime in place. During regimes I and D, fiat money holdings earn a gross rate of return of p_t/p_{t+1}. After the stabilization, I assume that coins depreciate at the same rate (δ) as raw gold, and that they circulate by weight.[11] Therefore, the rate of return on real balances during regime S is $(1\text{-}\delta)\, p_t/p_{t+1}$. For agents born at $T\text{-}1$ matters are more complicated, since they live through the transition to the gold standard, and hence will have their money balances (originally in fiat currency) redeemed by the government in gold coins at the exchange rate e. Thus, they face the gross real return: $e\, p_{T\text{-}1}/p_T$ on money balances they accumulate at $T\text{-}1$.

Once agents can freely mint and melt gold coins ($t \geq T$), the ability to convert goods into gold and conversely implies that

(6) $\quad p_t = 1/b\varepsilon, \qquad\qquad t=T,\ldots$

[11] This assumption is essentially innocuous.

The implied constancy of the price level in regime S allows the rate of return on real balances to be summarized as

$$Q_t = \begin{cases} p_t / p_{t+1}, & 1 \le t \le T-2 \\ e \cdot p_t / p_{t+1}, & t = T-1 \\ 1-\delta & t \ge T \end{cases} .$$

Lenders choose the composition of their portfolios between loans, l_t, real money balances, z_t, and real raw gold holdings, g_t, along with a consumption vector (c_1, c_2) to maximize (1) subject to the budget constraints

$$c_1 \le v - l_t - z_t - g_t$$

$$c_2 \le w_{t+1} + R_t l_t + Q_t z_t + (1\text{-}\delta) g_t.$$

and the reserve requirement[12]

$$z_t \ge (1\text{-}\phi)\cdot(z_t + l_t).$$

If $R_t > Q_t$ holds for all t, then the reserve requirement binds at each date. I focus throughout on this situation. In addition, if $\phi R_t + (1\text{-}\phi) Q_t > (1\text{-}\delta)$ holds at t, then raw gold is dominated as an asset, and it will not be held. During Regime S, $R_t > Q_t = 1\text{-}\delta$ implies that $\phi R_t + (1\text{-}\phi) Q_t > 1\text{-}\delta$ holds for $t \ge T$. For simplicity, I focus on equilibria where $\phi R_t + (1\text{-}\phi) Q_t > 1\text{-}\delta$ also holds for all $t \le T\text{-}1$. In this case the savings of lenders, which may be defined as $s_t \equiv l_t + z_t + g_t$, are allocated between assets according to

(7) $l_t = \phi \cdot s_t$

(8) $z_t = (1\text{-}\phi) \cdot s_t$

Since lenders hold a portfolio placing weights ϕ on loans and $(1\text{-}\phi)$ on reserves, the return on this portfolio is $\phi R_t + (1\text{-}\phi) Q_t$. Therefore, the optimal savings behavior of lenders is given by

(9) $$s_t = s(v, w_{t+1}, R_t, Q_t) = \frac{\beta}{1+\beta} v - \frac{1}{1+\beta} \cdot \frac{w_{t+1}}{\phi R_t + (1-\phi) Q_t} .$$

Borrowers

Borrowers of generation t choose a level of borrowing, b_t, and a consumption vector, (c_1, c_2), to maximize (1) subject to the budget constraints

$$c_1 \le b_t$$

$$c_2 \le y_{t+1} - R_t b_t.$$

The solution to this problem defines the following demand for consumption loans:

[12] See Wallace (1981), Sargent and Wallace (1985), and Smith (1991, 1994) for a discussion of how this condition, here imposed directly on the asset composition of lenders, can be regarded as a reserve requirement against the liabilities of an intermediary.

(10) $b_t = b(y_{t+1}, R_t) = \frac{1}{1+\beta}\frac{y_{t+1}}{R_t}$.

GENERAL EQUILIBRIUM

I concentrate on situations where the reserve requirement binds and no raw gold is held. Thus, in equilibrium

$R_t > \phi \cdot R_t + (1-\phi) \cdot Q_t > 1-\delta$

must obtain for all t. Second, the loan market must clear:

(11) $\mu \cdot b_t = (1-\mu) \cdot l_t = (1-\mu)\phi \cdot s_t \qquad \forall t$

Third, the supply of real balances must equal the demand for real balances, which is generated by the demand for reserves:

(12) $\frac{M_t}{p_t} = (1-\mu) z_t = (1-\mu)(1-\phi) s_t \qquad \forall t$

Finally, the government budget constraint must be satisfied at each date. At t=1, this requires

(13) $M_0 = M_1 - p_1 \cdot \gamma$.

If γ_t denotes real government expenditures at t, then the government budget constraint for t=2,...,T-1 can be summarized as

(14) $\frac{M_t}{p_t} - \frac{M_{t-1}}{p_{t-1}} Q_{t-1} = \gamma_t = \begin{cases} \gamma & \text{for } t = 2,...,T_1 \\ 0 & \text{for } t = T_1+1,...,T-1 \end{cases}$.

In addition, for $t = T_1$+1, ..., T-1, (3) describes the evolution of the government's gold reserves. At t=T the government faces the budget constraint:

(15) $\frac{M_{T-1}}{p_{T-1}} Q_{T-1} = a_T$.

From t=T+1 the government plays only a passive role, and its budget constraint is trivially satisfied.

The general equilibrium of the model under the assumptions of binding reserve requirement and no raw gold holdings is described by the savings function (9), the demand curve for consumption loans (10), the loan and money market clearing conditions (11) and (12), the description of the government's budget constraint (13), (14) and (15) with a_T defined by (4), and the no arbitrage condition (6).

It will now be convenient to introduce the notation V=(1-μ)·v, W_{t+1}=(1-μ)·w_{t+1} and Y_{t+1}=μ·y_{t+1} for the aggregate post-tax endowments of borrowers and lenders. Similarly, let Y=μ·y, W=(1-μ)·w, $\hat{Y} = \mu \cdot \hat{y}$ and $\hat{W} = (1-\mu) \cdot \hat{w}$. Using (9), (10), and these definitions, (11) can be rewritten as

(16) $\frac{Y_{t+1}}{R_t} = \phi\beta V - \frac{\phi W_{t+1}}{\phi R_t + (1-\phi)Q_t}$.

Rearranging terms in (16) yields the equilibrium relationship between the rate of return on real balances and the real interest rate as

(17) $Q_t = \frac{\phi}{1-\phi}\left(\frac{W_{t+1}}{\phi\beta V - Y_{t+1}/R_t} - R_t\right)$.

Combining (11) and (12), and using the same definitions results in the following expression for aggregate real balances

(18) $\frac{M_t}{p_t} = \frac{(1-\phi)}{(1+\beta)\phi} \cdot \frac{Y_{t+1}}{R_t}$.

Consequently, the set of equilibrium conditions reduces to (13), (14), (15), (17), (18), along with (4) and (6).

In order to guarantee that the reserve requirement will be binding, I henceforth assume that — in Gale's (1973) terminology — this is a "classical case" economy, so that:

(A.1) $\beta V < W + Y$.

It will also be necessary to assume that government expenditures during regime *I* are not too large:

(A.2) $(1+\beta)\gamma < Y$.

The role of (A.2) will be made more precise in the next section.

3. Equilibrium

In this section I first describe the construction of an equilibrium path assuming that all elements of the stabilization plan — the postwar fiscal policy (η^B and η^L), the date of gold resumption (T) and the par of exchange (e) — are given. I show that the relationship between the policy variables η^B, η^L, T and e must satisfy a restriction in order for the stabilization program to be internally consistent and an equilibrium to exist. In the second part of this section I investigate the nature of this relationship.

CONSTRUCTION OF AN EQUILIBRIUM PATH

It will be useful to proceed backwards in time, starting with the stabilization, and focus on the determination of the real interest rate (R_t), the rate of return on currency (Q_t) and the price level (p_t). Once these sequences are determined, positive values for the remaining

endogenous variables of the model, b_t, s_t, l_t and z_t, can be obtained from equations (10), (9), (7) and (8). The endogenous equilibrium money supply can be calculated as $M_t=(1-\mu)z_t \cdot p_t$. It is also important to check that along the constructed path both currency and raw gold are dominated in rate of return.

Stabilization

Access of agents to the unrestricted minting and melting of gold implies that $p_t=1/b\varepsilon$ and $Q_t = 1-\delta$ for all $t \geq T$. The equilibrium sequence $\{R_t\}_{t=T}^{\infty}$ is then determined by the credit market clearing condition

$$(19) \quad \frac{Y}{R_t}=\phi\beta V-\frac{\phi W}{\phi R_t+(1-\phi)(1-\delta)}, \quad t \geq \mathrm{T}.$$

Evidently, (19) yields a unique equilibrium interest rate $R_t=R_S^*$ for all $t \geq T$.

Note that the money supply sub-sequence, $\{M_t\}_{t=T}^{\infty}$, is also endogenous: it satisfies

$$(20) \qquad M_t=\frac{1-\phi}{1+\beta}\left[\beta V-\frac{W}{\phi R_s^*+(1-\phi)(1-\delta)}\right]\frac{1}{b\varepsilon}.$$

As long as the rate of depreciation of gold is positive, (20) implies positive minting of coins for each $t>T$.

Gold Resumption

At T-1, equations (4), (15), (17) and (18) imply that

$$(21) \quad R_{T-1}=\frac{\hat{Y}}{\phi\beta V}\cdot\frac{\hat{Y}+\hat{W}+(1+\beta)\Delta\theta}{\hat{Y}+(1+\beta)\Delta\theta},$$

and

$$(22) \quad Q_{T-1}=\frac{(1+\beta)\Delta\theta}{(1-\phi)\beta V}\cdot\frac{\hat{Y}+\hat{W}+(1+\beta)\Delta\theta}{\hat{Y}+(1+\beta)\Delta\theta}$$

The time T-1 price level, p_{T-1}, can be calculated from $p_{T-1}=p_T \cdot Q_{T-1}/e$.

The Fiat Currency System

Equations (14), (17) and (18) can be used to derive R_{t-1} as a function of R_t for t=2, ...,T-1. In particular,

$$(23) \quad R_{t-1}=\frac{Y_t}{\phi\beta V}\left(1+\frac{W_t\phi R_t}{(Y_t-(1+\beta)\gamma_t)\phi R_t+(1-\phi)Y_{t+1}}\right):=J_t(R_t), \qquad t=2, ..., T-1.$$

Note that (23) is not an autonomous difference equation for the entire period t=2, …, T-1, because of the time-dependence of γ_t, Y_t, Y_{t+1} and W_t. In fact, when gold resumption is delayed (i.e. $T>T_1+1$), three different consecutive laws of motion apply as special cases of (23). These laws of motion are given in Table 1. During the regime of wartime inflation (for t=2, ...,

T_1-1), J_t is a time invariant function, denoted by J_I. Similarly, during the postwar deflationary regime (for t=T_1+1, ..., T-1), J_t is also time invariant, and it is denoted by[13] $\hat{J}_D$. Finally, at the transition between the two regimes $J_{T_1}=\tilde{J}_I$. When gold resumption immediately follows the end of the war (T=T_1+1), the postwar deflationary period is missing. In this case $R_{T_1-1}=\tilde{J}_I(R_{T_1})=\tilde{J}_I(R_{T-1})$ obtains, and $R_{t\text{-}1}=J_I(R_t)$ for $t<T_1$.

Figure 2 depicts the generic function J_t. Assumption (A.2), together with the timing of taxes and government expenditures, implies that $Y_t>(1+\beta)\cdot\gamma_t$ holds for all t. Consequently, the graph of J_t is a hyperbola with the properties depicted in Figure 2. Moreover, it can be verified that the condition $R=J_t(R)$ has a unique positive solution for each t. This "steady state" (if the relevant regime were to remain in place indefinitely) is stable in the backward dynamics of (23), and unstable in the forward dynamics. Let R_I^* and $\hat{R}_D^*$ denote the fixed points: $R_I^*=J_I(R_I^*)$ and $\hat{R}_D^*=\hat{J}_D(\hat{R}_D^*)$ respectively.

We can now obtain $R_{T-2}=\hat{J}_D(R_{T-1})$, $R_{T-3}=\hat{J}_D(R_{T-2})=\hat{J}_D^2(R_{T-1})$, and so on, so that $R_{T_1}=\hat{J}_D^{T-T_1-1}(R_{T-1})$. Similarly, $R_{T_1-1}=\tilde{J}_I(R_{T_1})$, and $R_{T_1-1-i}=J_I^i(R_{T_1-1})$ for i=1, …, (T_1-2). This construction is illustrated in Figure 3.

Once the sequence $\{R_t\}_{t=1}^{T-2}$ is determined, (17) gives Q_t as

$$(24)\quad Q_t=\frac{\phi}{1-\phi}\cdot\left[\frac{\hat{W}}{\phi\beta V-\hat{Y}/R_t}-R_t\right]:=\hat{Q}(R_t) \qquad \text{for } t=T_1,\ \ldots,\ T\text{-}2,$$

and

$$(25)\quad Q_t=\frac{\phi}{1-\phi}\cdot\left[\frac{W}{\phi\beta V-Y/R_t}-R_t\right]:=Q(R_t) \qquad \text{for } t=1,\ \ldots,\ T_1\text{-}1.$$

The values of $\{p_t\}_{t=1}^{T-2}$ can be determined recursively using $p_{t\text{-}1}=Q_{t\text{-}1}\,p_t$.

By construction, the sequence of real interest rates obtained in the above described manner (together with the implied values of the other endogenous variables) satisfy the market clearing conditions for all t, and the government budget constraint for t>1. The last remaining condition of an equilibrium is that the government budget constraint at t=1 must also be met. Using (13), (18), (6) and the definition of Q_t, this budget constraint requires that

$$(26)\quad M_0=\left(\frac{(1-\phi)Y}{(1+\beta)\phi R_1}-\gamma\right)\cdot\left(\prod_{t=1}^{T-1}Q_t\right)\cdot\frac{1}{eb\varepsilon}.$$

Thus, η^B, η^L, T, and e must be chosen in accordance with (26) in order for an equilibrium to exist.

[13] The caret in the notation $\hat{J}_D$ serves as a reminder that the law of motion during this regime depends on the taxation scheme in place. In Section 6, taxation during regime D is eliminated.

Finally, it remains to verify that along the equilibrium path the reserve requirement binds and raw gold is not held by agents. The following proposition gives conditions under which these requirements are true, and therefore there is an equilibrium, as just constructed.

Proposition 1.

Assume (A.1) and (A.2) hold. Then the model has a unique equilibrium with currency and raw gold dominated in rate of return, if the elements of the stabilization program are chosen so that (26) is satisfied and the following conditions hold:

(27) $\gamma < \frac{(1-\phi)\beta V}{1+\beta} \cdot \frac{Y}{Y+W}$,

(28) $\phi(1+\beta)\Delta\theta < (1-\phi)\hat{Y}$,

and

(29) $\gamma < \frac{(1-\phi)\hat{Y}}{\phi(1+\beta)\hat{R}_D^*}$.

Condition (27) restricts government expenditures to be potentially sustainable forever. Higher values of γ would be consistent only with a short duration of regime *I*. Condition (28) requires that taxes not be too high. Higher taxes would result in a more favorable rate of return on currency, and the reserve requirement would not be binding during regime *D*. Finally, (29) is a technical condition, required at the transition between regimes *I* and *D*.[14] The proof of Proposition 1 is given in the Appendix. It shows that (27), (28) and (29) imply that the equilibrium path constructed above satisfies $R_t > Q_t > 0$ and $\phi R_t + (1-\phi)Q_t > 1-\delta$.

CONSTRAINTS ON POLICY CHOICES

Thus far I have described the equilibrium values of prices and interest rates for given choices of government policy. In order for an equilibrium to exist, these policy choices must be consistent with each other, i.e. they must satisfy the restriction imposed by (26). Now I investigate the nature of this restriction.

Define Ψ to be the product of the first two terms on the right-hand side of (26), so that

$$\Psi(\eta^B, \eta^L, T) := \left(\frac{(1-\phi)Y}{(1+\beta)\phi R_1} - \gamma \right) \cdot \left(\prod_{t=1}^{T-1} Q_t \right),$$

[14] The endogenous variables in (26) and $\hat{R}_D^*$ in condition (29) are thought of as a function of parameters and the policy variables η^B and η^L.

where the notation reflects the fact that R_1 and Q_t depend on T, η^B and η^L through the previous construction. It is easy to verify that Ψ does *not* depend on e. Then (26) can be rewritten as

(30) $eb\varepsilon M_0=\Psi(\eta^B, \eta^L, T)$.

The term $eb\varepsilon M_0$ represents the real value of resources at T which are required to redeem the initial fiat money supply outstanding at the beginning of period one. The total value of resources used for redemption, a_T, must, of course, also cover the cost of redeeming the additional fiat currency issued during the war.

Clearly, (30) can always be solved for e.[15] Thus, for any choice of fiscal policy and redemption date, a suitable exchange rate can be found at which a perfect foresight equilibrium exists (provided that the inequalities (27), (28) and (29) are also met). The historically more relevant interpretation, however, is that the government desires to return to gold at a given exchange rate (say, at pre-war par) at some pre-announced future date. In this case, equation (30) restricts the required postwar government surpluses.

Equation (30) suggests that different combinations of η^B and η^L — i.e. different allocations of the tax burden between borrowers and lenders — can be consistent with gold resumption at a given date and parity[16]. Moreover, $\partial\Psi/\partial\eta^B>0$ and $\partial\Psi/\partial\eta^L>0$ do not hold in general. This is due to the complex effects that the allocation of the tax burden has on interest rates, and through this channel on all important variables of the economy. To simplify the analysis, I first investigate the relationship between the various elements of the stabilization program under the somewhat neutral and algebraically tractable assumption that $\eta^B=\eta^L=:\eta$. This assumption tends to make Ψ increasing in η, so that there is a unique (constant) postwar budget surplus that permits gold resumption as scheduled. By contrast, when η^B is high compared to η^L, Ψ may not be monotonous in η^B or η^L, so that there is more than one postwar tax policy that is consistent with the gold resumption policy. The Laffer curve considerations that arise under the latter scenario are investigated in Section 5.

Accordingly, suppose that $\eta^B=\eta^L=\eta$. Then $\theta=\eta(Y+W)$, and the following conditions guarantee the existence of an equilibrium.

[15] It is easy to show using (27) that $\Psi(\eta^B, \eta^L, T)>0$.

[16] This allocation was the focus of Keynes's (1940) proposal for deficit finance in World War II.

Corollary 1 to Proposition 1.

Assume $\eta^B=\eta^L=\eta$ and (A.1) holds. An equilibrium with $R_t>Q_t$ and $\phi R_t+(1-\phi)Q_t>1-\delta$ exists, if the following conditions are satisfied:

(31) $\gamma<\gamma_{\max}:=\frac{(1-\phi)\beta V}{1+\beta}\cdot\frac{Y}{Y+W}$

(32) $\eta<\eta_{\max}:=\frac{(1-\phi)Y}{(1-\phi)Y+\phi(1+\beta)\Delta(Y+W)}$

This corollary can be verified by showing that (31) and (32) together imply (27), (28) and (29). The quantity $\gamma_{\max}$ represents the highest level of government deficits that would be sustainable forever, while is the maximum tax rate consistent with an equilibrium where the reserve requirement is binding.

When the government levies equal proportional taxes on all second period income, Ψ is most likely to be an increasing function of these taxes. Thus, there is a unique equilibrium tax rate that allows the government to resume gold convertibility as scheduled, with a higher intended parity requiring higher taxes. A sufficient condition for this outcome to occur is given in the following proposition, which is proved in the Appendix.

Proposition 2.

Assume $\eta^B=\eta^L=\eta$, and (A.1), (31) and (32) hold. If

(33) $\phi>\frac{W}{4Y+5W}$

then $\Psi(\eta, \eta, T)$ is an increasing function of η.

Inequality (33) is a relatively weak condition: whenever $\phi>0.2$ (the reserve requirement, $1-\phi$, is less than 80%), it is trivially satisfied. Moreover, (33) is merely a sufficient condition. Extensive numerical simulations indicate that $\partial\Psi/\partial\eta>0$ holds even when (33) is violated, so long as (A.1), (31), and (32) are satisfied.

The dependence of Ψ on T can also be investigated numerically for the case when $\eta^B=\eta^L=\eta$. For a typical parameter combination, Figure 4 shows the graph of $\Psi(\eta, \eta, T)$ as a function of η and for various values of T. From Figure 4 it is apparent that Ψ is also increasing in T. Thus, for a given tax rate, the longer the postwar period of reserve accumulation, the higher the redemption parity can be. And, a given redemption rate can be achieved by raising high taxes and resuming quickly, or by waiting longer and spreading the tax burden across more generations.

4. Implications of different stabilization plans

In this section I examine the implications of alternative choices regarding the elements of a stabilization plan. For tractability, I maintain the assumption that taxes are proportional to the old age endowments of borrowers and lenders, that is $\eta^B=\eta^L=\eta$. I think of the government as choosing a combination of the resumption date (T) and parity (e), and using (30) to determine the necessary rate of taxation that will support this plan.[17] I describe how alternative stabilization plans affect the dynamic path of the economy and the welfare of various agents.

QUALITATIVE FEATURES OF EQUILIBRIUM PATHS

In order to describe qualitative features of equilibrium paths, it is useful to begin by examining the relationships between the real rate of interest on loans at t, R_t, and other endogenous variables.

Equations (24) and (25) give Q_t (the inverse rate of inflation) as a decreasing function of R_t. Thus, within any period, the higher the rate of interest on loans, the higher is the rate of inflation. It will also be useful to know the gross rate of interest on loans that is consistent with an unchanging price level. During t=1, …, T_1-1, let the solution to the condition $Q_t=Q(R_t)=1$ be denoted by $R_t=R_D^*$. During $t=T_1$, … , T-1, the solution to $Q_t=\hat{Q}_t(R_t)=1$ can be shown to be $R_t=\hat{R}_D^*$. Moreover, it is easy to show that $\hat{R}_D^*<R_D^*<R_I^*$. It follows from these observations that, for t=1, …, T_1-1, the price level is rising between t and t+1 iff $R_t>R_D^*$. Similarly, for $t=T_1$, …, T-1, the price level is rising between t and t+1 iff $R_t>\hat{R}_D^*$.

Typical historical experience with gold suspension and subsequent resumption involves a wartime inflation, followed by a postwar deflation leading up to the reestablishment of a gold standard. This pattern will be replicated by the model iff

(34) $R_{T-1}<\hat{R}_D^*<R_D^*<R_{T_1-1}$

holds. Indeed, if (34) is satisfied, then $R_{T-1}<R_{T-2}<\ldots<R_{T_1}$ obtains, so that not only is there a postwar deflation, but this deflation increases in magnitude as gold resumption nears. Similarly, if $R_{T_1-1}>R_D^*$ then there is a wartime inflation. If $R_1<R_I^*$ then $R_{T_1-1}<R_{T_1-2}<\ldots<R_1$ holds: in this case the loan rate and the rate of inflation fall as the end of the war approaches. On the other hand, if $R_1>R_I^*$ then $R_1<R_2<\ldots<R_{T_1-1}$, and inflation accelerates as the war draws to a conclusion.

[17] Since under the assumption $\eta^B=\eta^L=\eta$ the function $\Psi(\eta,\eta,T)$ is increasing both in η and in T, a higher parity or earlier resumption requires higher taxes.

Which time pattern of interest rates and inflation rates obtain depends heavily on the policy choices made with respect to gold resumption. The following proposition describes some possibilities, which are also illustrated diagrammatically in Figure 5.

Proposition 3

Assume $\eta^B=\eta^L=\eta$.

i. If $T=T_1+1$, then there is a value, $\eta_1(\gamma)$, such that $R_{T_1-1}<R_I^*$ iff $\eta>\eta_1(\gamma)$. Moreover, $\eta_1(\gamma)$ is a decreasing function of γ with $0<\eta_1(0)<\eta_{max}$, and $\eta_1(\gamma_{max})=0$.

ii. If $T=T_I+2$, then there are two values, $\eta_1(\gamma)$ and η_2, such that $R_{T_1-1}<R_I^*$ iff $\eta>\eta_1(\gamma)$ and $R_{T-1}<\hat{R}_D^*$ iff $\eta>\eta_2$. Moreover, $\eta_1(\gamma)$ is a decreasing function of γ, $0<\eta_1(0)<\eta_2<\eta_{max}$, and $\eta_1(\gamma_{max})<0$.

iii. If $T\rightarrow\infty$, then there is a critical value, η_2, satisfying $0<\eta_2<\eta_{max}$, such that $R_{T-1}<\hat{R}_D^*$ iff $\eta>\eta_2$.

If the return to gold is immediate ($T=T_1+1$), then a high redemption parity (that is, high taxes) imply that the economy will experience a decelerating rate of wartime inflation. As gold resumption is delayed ($T=T_1+2$) the same statement is true. Moreover, whether there is a postwar inflation or a postwar deflation also depends on the exchange rate that will apply when gold convertibility is resumed. Finally, as gold convertibility is delayed indefinitely ($T\rightarrow\infty$), only a decelerating wartime inflation can be observed. A postwar deflation results under high government budget surpluses, while low surpluses imply that the inflation continues after the war ends.

From the perspective of lenders' welfare, it will also be useful to understand the relationship between the rate of return on savings and the rate of return on loans. the rate of return on savings satisfies

$$(35)\quad \phi R_t+(1-\phi)Q_t=\frac{\phi W_{t+1}}{\phi\beta V-Y_{t+1}/R_t}-1\,.$$

Thus, as R_t rises, the rate of return on savings falls. A further implication of this relationship is that the higher the rate of return on loans, the larger is the "wedge" between the marginal rates of substitution of borrowers and lenders.

Finally, under the assumption that borrowers and lenders are taxed at the same rate, the following proposition states a result about how this rate of postwar taxation affects loan rates, "deposit" rates, the rate of inflation and the initial price level.

Proposition 4

Assume $\eta^B=\eta^L=\eta$. For a given T, the following relationships hold:

$dR_t/d\eta<0$, for $t=1, \ldots, T-1$

$dQ_t/d\eta>0$, for $t=1, \ldots, T_1-1, T-1$

$d(\phi R_t+(1-\phi)Q_t)/d\eta>0$, for $t=1, \ldots, T_1-1, T-1$, and

In addition, if (33) holds[18] then

$dp_1/d\eta<0$.

Thus, for a given delay before resumption, a higher postwar par of exchange — which require higher postwar surpluses — imply lower interest rates at all dates, higher deposit rates and lower inflation rates during the war, and a lower initial price level. By implication, the higher the resumption parity, the lower the price level at every date during the war.

WELFARE

As discussed earlier, the issues of when to return to gold after a war, and at what rate of exchange, have historically been contentious ones. This is not surprising in light of the fact that different generations, and different groups within a generation, can easily be affected in markedly different ways by a particular policy choice. Indeed, Keynes (1940) viewed issues of wartime finance as primarily being about which groups would bear what burden of the costs of the war.

Proposition 4 implies some immediate results about how the welfare of wartime generations depends on gold resumption. Since wartime generations are not taxed to accumulate gold reserves, it is clear that — for a given date of resumption — wartime generations prefer a high postwar rate of exchange. In particular, higher values of e imply higher values of η. This, in turn, means lower loan rates (which are desired by borrowers), higher deposit rates (which are desired by lenders), and a lower initial price level (which is desired by the initial old). Thus all wartime groups and generations have coincident interests with respect to the postwar par of exchange. From this perspective it is not surprising that the typical announced policy during a suspension of gold convertibility is that gold convertibility will be resumed without a devaluation (that is, at the prewar par of exchange).

[18] As in the case of Proposition 2, condition (33) is only a sufficient condition. Numerical results suggest that $dp_1/d\eta<0$ s true for all ϕ.

Once the government starts to run surpluses for the purpose of accumulating reserves, the allocation of the tax burden can interact with different rates of return in a more complicated way. In addition, the implications of the choice of T for the welfare of agents is also more difficult to characterize analytically. Instead, it is possible to use numerical simulations of the model. In order to summarize the welfare effects of policy choices for various groups of agents it is also useful to introduce a social welfare index.

Social welfare function

The initial old agents only consume in a single period, so that their utility just depends on the value of their initial money holdings. Therefore I set $u_0=c_0=M_0/p_1$. The utility of all other agents, given by (1), is also measured in units of the real good and is interpreted as consumption averaged across their two periods of lifetime.

Within each young generation, I aggregate the utility of borrowers and lenders using a utilitarian welfare function that weights the utility of the two groups according to their shares in the population. Thus, if u_t is the utility of the generation born at t,

$$u_t = \mu \cdot u_t^B + (1-\mu)\cdot u_t^L = \mu \cdot u(c_1^B, c_2^B) + (1-\mu)\cdot u(c_1^L, c_2^L)$$

where the superscripts B and L denote borrowers and lenders respectively.

I also assume that the social welfare function discounts the utility of future generations at the rate $\rho \in (0,1]$. Moreover, I will want to talk about the aggregate welfare of generations born at $t=T_1$, , T-1. Since the set of generations in question will vary as the time of resumption (T) is varied, it is more appropriate to talk about a weighted average, rather than a weighted sum, of utilities. Thus, if $U[t_1+1,\ t_2]$ denotes the aggregate welfare of members of generation $t_1+1, \ldots, t_2$, I set

$$(36)\quad U[t_1+1,\ t_2] = \frac{1-\rho}{1-\rho^{t_2-t_1}} \cdot \sum_{t=t_1+1}^{t_2} \rho^{t-t_1-1} \cdot u_t$$

In the limit, as $\rho \uparrow 1$, this is just an arithmetic average across a finite number of generations. And for $\rho<1$, we have

$$(37)\quad U[t_1+1,\ \infty] = (1-\rho) \cdot \sum_{t=t_1+1}^{\infty} \rho^{t-t_1-1} \cdot u_t\ .$$

Wartime generations

As mentioned earlier, the initial old, wartime borrowers and wartime lenders all unanimously prefer high gold parity for any given date of resumption. Numerical simulations establish that regarding the date of resumption itself, the preferences of these agents are also

aligned. Whether they desire quick or slow resumption, however, depends on the value of e. Figure 6 shows a typical plot of the aggregate welfare of generations 1, ..., T_1-1 and the initial old against $eb\varepsilon M_0$ for various values of T.[19] Clearly, for a given value of e, rapid resumption is desirable only if e is sufficiently high. Nevertheless, agents alive during the wartime overall prefer a combination of rapid resumption and high parity.

Postwar generations

The welfare of postwar generations is influenced by the stabilization policy not only through the rates of return on different assets but also through the their tax obligations. Hence it will no longer typically be the case that different members of postwar generations have common attitudes regarding resumption policies. In particular, the interests of borrowers versus lenders within the same generation, as well as the interests of agents born at different times after the war, can potentially diverge.

Extensive numerical simulations indicate that it is useful to break postwar generations into two groups. The first group consists of agents born at $T_1, \ldots, T$-2. These agents are required to pay taxes in preparation for the resumption, but they are not alive at the time when gold convertibility is actually re-established. The second group consists of members of generation T-1, who are responsible for executing the stabilization.

My calculations indicate that borrowers and lenders in generations $T_1, \ldots, T$-2 all prefer low values of e and high values of T. The aggregate welfare of these agents as a function of the policy choices is illustrated in Figure 7.[20] Thus, in contrast to wartime generations who desire a speedy resumption at old par, postwar generations are interested in delaying the stabilization and devaluing the currency.

Stabilizations nevertheless often do take place eventually. In the context of the model this is explained by the fact that agents born at T-1 might have a stronger interest in the reestablishment of the gold standard than earlier postwar generations. Extensive numerical experiments indicated that two different scenarios can be distinguished regarding the effect of e on the welfare of T-1 borrowers and T-1 lenders. The first scenario, illustrated in Figure 8, corresponds to the situation when borrowers are either wealthy or numerous compared to lenders. Figure 9 represents the opposite case. Both of these figures suggest that if resumption is scheduled to take place relatively early, then borrowers and lenders within generation T-1 have opposing views about the desirable par of exchange. Delaying resumption, on the other hand, helps build consensus between these two groups so that they can both agree on returning to

[19] Qualitatively identical results can be obtained if each homogenous subgroup is examined separately.
[20] Again, each homogenous subgroup behaves qualitatively similarly to the average.

gold at a high parity. A comparison of Figure 8 and Figure 9 also indicates that, loosely speaking, the group with higher aggregate wealth is more likely to be interested in devaluing the currency, while the group with lower aggregate wealth is more inclined to honor the wartime promise of resumption at par.

These results are in remarkable agreement with the historical record. When abandoning the gold standard, governments often promised to resume gold payments quickly and at prewar par. By contrast, postwar periods exhibited fierce political fights between holders of the government debt who were trying to preserve the real value of this debt by legislating speedy resumption at the promised pre-war par, and those who would benefit from the annulment of this old debt by an inflationary erosion caused by a resumption at a low parity. As a result, resumption is often delayed well beyond the date originally promised (or implied) by the government.

5. Alternative tax policies and the possibility of multiple equilibria

The results of the previous sections, derived under the assumption that taxation takes the form of an equal fraction of the two types of agents' old age endowments, are not robust to changes in the way taxes are levied. A particularly interesting phenomenon, which can arise under some alternative specifications, is the possibility that $\Psi(\eta^B, \eta^L, T)$ is no longer monotonic in the two tax rates. When this happens, a return to a gold standard at a given fixed time and at a given fixed redemption rate is possible with different fiscal measures. In effect, Laffer-curve considerations arise.

Extensive numerical simulations indicated that multiple equilibrium fiscal policies generally arise when the tax burden is most skewed towards borrowers. Therefore, to simplify the discussion, I will examine the implications of the assumption that $\eta^L=0$, so that only borrowers are ever taxed.[21] The following result states some conditions under which an equilibrium exists with this structure of taxation.

[21] If the absolute tax amount payable by the two types of agents is equal, multiple equilibria can be found under some extreme parameter settings where borrowers have very low and lenders have very high old age endowments, and the number of borrowers is high relative to the number of lenders. These parameter combinations correspond to a high implied tax rate on borrowers and a low implied tax rate on lenders. However, focusing on the most extreme case that only borrowers are taxed substantially simplifies the presentation.

Corollary 2 to Proposition 1

Assume $\eta^L=0$ and (A.1) holds. An equilibrium with $R_t>Q_t$ and $\phi R_t+(1-\phi)Q_t>1-\delta$ exists if

$\gamma<\gamma_{max}$, as defined in (31),

(38) $\eta^B<\eta^B_{max}:=\dfrac{1-\phi}{1-\phi+\phi(1+\beta)\Delta}$, and

(39) $\eta^B<\bar{\eta}(\gamma):=1-\dfrac{(1+\beta)\gamma}{Y}\left(\dfrac{W}{(1-\phi)\beta V-(1+\beta)\gamma}-1\right)$.

The observations that under $\eta^L=0$ (38) is equivalent to (28) and (39) is equivalent to (29) prove the corollary.

Figure 10 depicts, for a set of parameters that satisfy the conditions of the corollary, the possibility that Ψ may be non-monotonic in η^B. This situation will obtain when lenders have low first period endowments, and, relative to borrowers, high second period endowments. In addition, government expenditures are close to the maximum value which is consistent with the existence of an equilibrium. For other combinations of endowments or for lower government expenditures $\Psi(\eta^B, 0, T)$ is increasing both in η^B and T, similarly to the case when $\eta^B=\eta^L$. Note also that Laffer curve considerations do not emerge when gold convertibility is immediate after the war.[22]

When the configuration illustrated in Figure 10 obtains, then for some values of T, the Ψ function is first increasing and then decreasing in η^B. This allows for two different values of η^B to be consistent with the same resumption date and parity. It is also the case that Ψ is no longer monotonic in T: extending the postwar period of reserve accumulation may not increase the parity at which gold convertibility can be resumed.

When multiple fiscal policies can achieve resumption at the same time and same parity, it is interesting to compare the dynamics implied by different tax rates. Figure 11 illustrates sample paths for the loan rate, deposit rate and the price level under high and low tax rates for an economy with multiple equilibria. Perhaps the most striking feature of this figure is the large difference between the two price level paths. Higher taxes under this scenario imply higher wartime inflation rates along with more severe postwar deflation.

One might imagine that an unnecessarily high rate of postwar taxation would have uniformly negative welfare consequences, particularly for borrowers who bear the tax burden.

[22] In fact, it is possible to show in general that in the special case when $T=T_1+1$, $\partial\Psi(\eta^B, \eta^L, T)/\partial\eta^B>0$ and $\partial\Psi(\eta^B, \eta^L, T)/\partial\eta^L>0$ for all values of η^B and η^L.

Figure 12 depicts the welfare of members of different groups and generations under the two policies. As the figure illustrates, borrowers and lenders have the same preferences with respect to the choice of postwar fiscal policy. However, not all generations prefer the apparently more efficient low rate of taxation. Indeed, members — even borrowers — of the transitional (T-1) generation prefer to be taxed heavily. This is due to the fact that the high rate of taxation exerts strong downward pressure on the rate of interest on loans, and some upward pressure on the rate of interest on deposits. In general, then, there is no presumption that a low postwar tax rate is Pareto superior to a high one when Laffer-curve phenomena arise.

6. Consequences of not accumulating gold reserves

For a successful resumption of gold convertibility at T, it is not necessary that the government accumulate gold reserves prior to T.[23] In this section I examine how the dynamic behavior of the economy is affected by the assumption that the resources required for redemption of the fiat money are raised only from taxes levied at T, without prior reserve accumulation taking place. I maintain the convention that these taxes are paid by the old agents. Accordingly, I assume that borrowers of generation T-1 pay a fraction ζ^B, while lenders of the same generation pay a fraction ζ^L of their old age endowment as taxes at time T. Thus, agents' post-tax endowments are

$$y_{t+1}=\begin{cases}\hat{y}=(1-\zeta^B)y & \text{for } t=T-1\\ y & \text{otherwise}\end{cases} \quad \text{and} \quad w_{t+1}=\begin{cases}\hat{w}=(1-\zeta^L)w & \text{for } t=T-1\\ w & \text{otherwise}\end{cases},$$

and total taxes collected at T satisfy $\tau=\zeta^B\cdot Y+\zeta^L\cdot W$.

The equilibrium of the model under this scenario can be derived in the same way as before: first take the elements of the stabilization plan as given and solve for the dynamic path of loan rates and other endogenous variables. Once this is done, it is then possible to examine the restrictions that fiscal policy must satisfy in order to achieve a given resumption date and parity.

The construction of an equilibrium proceeds as follows. Clearly, the post-stabilization course of events is not affected by the change in the government's policy of reserve accumulation. Thus, $R_t=R_S^*$ for $t\geq T$. Similarly, the government budget constraint at T, (15), remains unchanged except for the definition of a_T, which is now given by (5). Therefore, $R_{T\text{-}1}$ satisfies

[23] This is quite contrary to the discussion in Friedman and Schwartz (1963), Chapter 2.

$$R_{T-1}=\frac{\hat{Y}}{\phi\beta V}\cdot\frac{\hat{Y}+\hat{W}+(1+\beta)\tau}{\hat{Y}+(1+\beta)\tau}.$$

The laws of motion for R_t, $t = 1, \ldots, T\text{-}2$ are given in Table 2,[24] and represented diagrammatically in Figure 13. If we define

$$\Omega(\zeta^B, \zeta^L, T):=\left(\frac{(1-\phi)Y}{(1+\beta)\phi R_1}-\gamma\right)\cdot\left(\prod_{t=1}^{T-1}Q_t\right),$$

with R_1 and Q_t now determined as functions of ζ^B, ζ^L and T, then a consistent stabilization plan must also satisfy

(40) $\quad eb\varepsilon M_0 = \Omega(\zeta^B, \zeta^L, T).$

The following proposition states restriction under which an equilibrium with a binding reserve requirement and no raw gold storage exists.

Proposition 5

Assume (A.1) and (A.2) hold. Then the model has a unique equilibrium with currency and raw gold dominated in rate of return, if the elements of the stabilization program are chosen so that (40) is satisfied and the following conditions hold:

(41) $\quad \phi(1+\beta)\tau < (1-\phi)\hat{Y}$ and

(42) $\quad \gamma<\frac{(1-\phi)\beta V}{1+\beta}\cdot\frac{Y}{Y+W}.$

Equations (41) and (42) are the exact counterparts of (28) and (27), and have the same interpretation. Condition (29) is no longer needed, as a transition between inflation and deflation is guaranteed by (41) and (42). It is easy to construct a proof analogous to the proof of Proposition 1.

It is straight-forward to show that $\partial\Omega/\partial\zeta^B>0$ and $\partial\Omega/\partial\zeta^L>0$. Therefore, given any sensible rule of allocating total taxes between borrowers and lenders, there is a unique level of taxes that must be paid at T in order to achieve a given redemption parity at that time.[25] Moreover, if the date of gold resumption is fixed, a higher redemption rate requires higher taxes to be paid by at least one group of agents.

[24] The laws of motion in Table 2 apply when $T>T_1+1$. The case $T=T_1+1$ has been addressed already.

[25] Ω is increasing in τ, as long as the rule by which taxes are distributed between borrowers and lenders is such that $\partial\zeta^B/\partial\tau\geq 0$ and $\partial\zeta^L/\partial\tau\geq 0$, i.e. an increase in total taxes does not reduce the tax liability of either type of agent. Obviously, $\zeta^B=\zeta^L$, $\zeta^B=0$, $\zeta^L=0$, or $\zeta^B y=\zeta^L w$ are all examples of such a rule. The dynamic path of the economy, and as a result the total value of taxes, however, is still not completely independent of the distribution of these taxes.

The dependence of $\Omega(\zeta^B, \zeta^L, T)$ on T can be much different from the relationship between $\Psi(\eta^B, \eta^L, T)$ and T, which was observed earlier. Under the simplifying assumption that $\zeta^B=\zeta^L=:\zeta$, Figure 14 depicts the graph of $\Omega(\zeta, \zeta, T)$ for a set of parameter values that satisfy (41) and (42). Here, increasing the delay before the restoration of gold convertibility will increase the tax burden on the transitional generation if $eb\varepsilon M_0$ is low, but raise it if $eb\varepsilon M_0$ is high. Numerical simulations indicate that the typical situation is for Ω to be decreasing in T, except for very low values of $eb\varepsilon M_0$.

Some effects on other aspects of an equilibrium of higher tax rates paid by generation T-1 are summarized in the following proposition.

Proposition 6

Assume that only generation T-1 is taxed and T is given. Then for $i=B, L$

$dR_t/d\zeta^i<0$, for t=1, …, T-1

$dQ_t/d\zeta^i>0$, for t=1, …, T-1

$d(\phi R_t+(1-\phi)Q_t)/d\zeta^i>0$, for t=1, …, T-1, and

$dp_1/d\zeta^i<0$.

Thus, when the date of resumption is fixed, higher taxes (which allow for a higher redemption rate) imply that loan rates are lower, deposit rates are higher and inflation rates are lower at all dates prior to the stabilization. The proof of this proposition is analogous to but simpler than the proof of Pr0position 4, and therefore it is omitted.

Finally, moderate wartime inflation and an accelerating postwar deflation emerge — as depicted in Figure 13 — iff ζ^B and ζ^L are sufficiently large.

7. Summary and conclusions

Throughout much of history, episodes of large government spending (typically wartime episodes) have been accompanied by a temporary abandonment of a gold standard. Once government spending moderated, there were usually deflations leading up to a reestablishment of gold convertibility at a pre-announced rate of exchange and at a pre-announced date. Despite the repetition of this pattern in many times and places, consensus about the optimal way to accomplish the reestablishment of a gold standard has never been reached. Madison (1779) and Mitchell (1908) favored a rapid postwar return to gold. Politicians often argued that a return to a prewar par of exchange was a matter of national honor. Keynes (1931) thought it inadvisable

to generate postwar deflations. Keynes (1940) was also preoccupied with the distributional consequences of war finance and the associated conduct of monetary policy.

I have shown that the method and timing of resumption can have strong effects on the properties of dynamic equilibria. As Madison (1779) and Mitchell (1908) argued, the magnitude of wartime inflation — as well as the magnitude of the postwar deflation — can be increased by policies related to the date and parity at which gold convertibility is restored. I have also argued that — given the typical methods used to accumulate gold reserves in anticipation of resumption — all wartime generations will favor the policy of a rapid return to gold at pre-war par. In contrast, among postwar generations, many will favor delaying resumption and/or devaluing the currency, and borrowers and lenders will likely disagree about the appropriate policy. These findings agree well with the historical record. Governments making wartime promises to quickly resume gold payments after the war have rarely been harshly criticized for this, at least not until the end of the war. At that time, however, the nature of the most desirable stabilization plan usually became a contentious political issue.

We have also seen that Laffer curve considerations regarding postwar budget surpluses can arise — particularly when different segments of the population are taxed at different rates. Somewhat surprisingly, it need not be the case that unnecessarily high postwar rates of taxation are Pareto inferior.

The welfare findings indicate that there is likely to be a time inconsistency problem with all the stabilization plans I examined. In a potential extension of the model I would allow for the possibility that the government cannot bind itself to the return to gold at a specific date or par of exchange. This would force me to consider policies that are time consistent.

I have also abstracted from the possibility that different policy choices have different consequences for the level of real activity. Historically postwar deflations were typically associated with recessions. This is an issue I hope to explore in future work.

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Appendix

Proof of Proposition 1

<u>*Stabilization*</u> ($t \geq T$). We need to show that $R_S^* > 1-\delta$. This follows from (19) and (A.1).
<u>*Gold Resumption*</u> ($t=T$-1). Clearly, $Q_{T-1}>0$. $R_{T-1}>Q_{T-1}$ is equivalent to (28). In addition, $\phi R_{T-1}+(1-\phi)Q_{T-1}=(Y+W+((1+\beta)\Delta-1)\theta)/\beta V > 1-\delta$ by (A.1).
<u>*Fiat Currency System*</u> ($t \leq T$-2). We need to show for all $t \leq T$-2 that (a) $0<Q_t<R_t$ and (b) $\phi R_t+(1-\phi)Q_t>1-\delta$.
(a) The condition $0<Q_t<R_t$ is equivalent to

(43) $$\frac{Y_{t+1}+\phi W_{t+1}}{\phi\beta V}<R_t<\frac{Y_{t+1}+W_{t+1}}{\phi\beta V}$$

For future reference it will be convenient to define two intervals, I and $\hat{I}$, as

$$I=\left(\frac{Y+\phi W}{\phi\beta V},\ \frac{Y+W}{\phi\beta V}\right) \text{ and } \hat{I}=\left(\frac{\hat{Y}+\phi\hat{W}}{\phi\beta V},\ \frac{\hat{Y}+\hat{W}}{\phi\beta V}\right)$$

For $t=T_1, \ldots, T$-2, we have $Y_{t+1}=\hat{Y}$, $W_{t+1}=\hat{W}$, and

$$\min\{\hat{R}_D^*, R_{T-1}\}<R_t<\max\{\hat{R}_D^*, R_{T-1}\}.$$

Therefore, it is sufficient to show that $R_{T-1}\in\hat{I}$ and $\hat{R}_D^*\in\hat{I}$.
The condition $R_{T-1}\in\hat{I}$ is equivalent to $0<Q_{T-1}<R_{T-1}$, and has been shown already. The condition $\hat{R}_D^*\in\hat{I}$ follows from (A.1) and (28).
For $t=T_1$-1, we have $Y_{t+1}=Y$, $W_{t+1}=W$, and

$$\tilde{J}_I\left(\min\{\hat{R}_D^*, R_{T-1}\}\right)<R_{T_1-1}<\tilde{J}_I\left(\max\{\hat{R}_D^*, R_{T-1}\}\right).$$

Therefore, (43) holds for $t=T_1$-1 iff $\tilde{J}_I(R_{T-1})\in I$ and $\tilde{J}_I(\hat{R}_D^*)\in I$. The condition $\tilde{J}_I(R_{T-1})\in I$ is equivalent to

$$\frac{(1-\phi)\hat{Y}}{(1-\phi)Y+\phi(1+\beta)\gamma}=\tilde{J}_I^{(-1)}\left(\frac{Y+\phi W}{\phi\beta V}\right)<R_{T-1}<\tilde{J}_I^{(-1)}\left(\frac{Y+W}{\phi\beta V}\right)=\frac{(1-\phi)\hat{Y}}{\phi(1+\beta)\gamma},$$

which can be directly verified using (21), (27) and (A.1). The condition $\tilde{J}_I(\hat{R}_D^*)\in I$ is equivalent to

(44) $$\frac{(1-\phi)\hat{Y}}{(1-\phi)Y+\phi(1+\beta)\gamma}<\hat{R}_D^*<\frac{(1-\phi)\hat{Y}}{\phi(1+\beta)\gamma}.$$

The left hand inequality in (44) follows from (A.1) and (28). The right hand inequality in (44) is equivalent to (29). For most parameter choices, (29) follows from (28) and (27), but it is possible to construct examples when this is not the case.

For t=1, …, T_1-2, we have $Y_{t+1}=Y$, $W_{t+1}=W$, and

$$\min\{R_I^*, R_{T_1-1}\}<R_t<\max\{R_I^*, R_{T_1-1}\}.$$

Therefore, to prove (43) for t=1, …, T_1-2, it remains to show that $R_I^*\in I$. This is implied by (A.1) and (27).

(b) The condition $\phi R_t+(1-\phi)Q_t>1-\delta$ is equivalent to

(45) $Y_{t+1}(1-\delta)+\phi R_t(W_{t+1}-(1-\delta)\beta V)>0$.

If $W_{t+1}\geq(1-\delta)\beta V$, then (45) is trivially true. If $W_{t+1}<(1-\delta)\beta V$, then (45) is equivalent to

(46) $R_t<\dfrac{(1-\delta)Y_{t+1}}{\phi((1-\delta)\beta V-W_{t+1})}$.

For $t=T_1$, …, T-2, let $\hat{R}^U:=(1-\delta)\hat{Y}/\phi((1-\delta)\beta V-\hat{W})$. As in part (a), (46) can be established for all $t\in\{T_1, ..., T\text{-}2)$ by showing that $R_{T-1}<\hat{R}^U$ and $\hat{R}_D^*<\hat{R}^U$ Condition $R_{T-1}<\hat{R}^U$ follows from $\phi R_{T\text{-}1}+(1-\phi)Q_{T\text{-}1}>1-\delta$, and has already been shown. It is possible to demonstrate that (A.1) and (28) imply $\hat{R}_D^*<\hat{R}^U$

For t=1, …, T_1-1, let R^U:=(1-δ)Y/φ((1-δ)βV-W). Since (A.1) implies that $R^U>(Y+W)/(\phi\beta V)$, condition (46) is implied by (43) for all $t\in(1, T_1\text{-}1)$.

Proof of Proposition 2

Rewrite $\Psi(\eta,\eta,T)$ as follows:

(47) $$\Psi(\eta,\eta,T)=\left(\frac{M_1}{p_1}-\gamma\right)\cdot\prod_{t=1}^{T_1-1}Q(R_t)\cdot\prod_{t=T_1}^{T-2}\hat{Q}(R_t)\cdot Q_{T-1}$$

From (18) and the constancy of M_t and of Y_{t+1} during $t=T_1$, …, T-1 it is clear that $\hat{Q}(R_t)=R_t/R_{t+1}$. Thus,

$$\prod_{t=T_1}^{T-2}\hat{Q}(R_t)Q_{T-1}=R_{T_1}\cdot\frac{Q_{T-1}}{R_{T-1}},$$

and (47) becomes

(48) $$\Psi(\eta,\eta,T)=\left(\frac{M_1}{P_1}-\gamma\right)\left(\prod_{t=1}^{T_1-1}Q(R_t)\right)\left(R_{T_1}\frac{\eta}{1-\eta}\right)\cdot c$$

with $c=\phi(1+\beta)\Delta(Y+W)/((1-\phi)Y)$. It is sufficient to show that the three non-constant terms in (48) are all increasing in η. By Proposition 4, $dR_1/d\eta<0$. Thus, from (18), $d(M_1/p_1)/d\eta>0$. Similarly, $dR_t/d\eta<0$ for t=1, ..., T_1-1 together with $\partial Q(R_t)/\partial R_t<0$ imply that $dQ(R_t)/d\eta>0$ for t=1, ..., T_1-1.

Define $X_t = R_t \frac{\eta}{1-\eta}$ for t = $T_1,...,T$-1. The claim that $dX_{T_1}/d\eta>0$ can be shown by the following induction argument. Using the relationship $R_t = \hat{J}_D(R_{t+1})$ for t=$T_1,...,T$-2, X_t can be written as

$$X_t = H(X_{t+1},\eta) := \frac{1}{\phi\beta V}\left[Y\eta + \frac{W}{\frac{1}{\eta}+\frac{1-\phi}{\phi(1-\eta)X_{t+1}}}\right]$$

It is easy to directly verify that $dX_{T-1}/d\eta>0$. In addition if

(49) $\frac{\partial H(X_{t+1},\eta)}{\partial X_{t+1}}>0$

and

(50) $\frac{\partial H(X_{t+1},\eta)}{\partial \eta}>0$

hold then $dX_{T_1}/d\eta>0$ follows from the chain rule by induction. Clearly, (49) holds. Forming the appropriate partial derivative and rearranging yields that (50) is equivalent to $q(X_{t+1}) > 0$, where the quadratic function $q(X_{t+1})$ is defined as

$$q(X_{t+1}) = X_{t+1}^2\left[\phi^2(1-\eta)^2(Y+W)\right] + X_{t+1}\left[\phi\eta(1-\phi)(2(1-\eta)Y-\eta W)\right] + \eta^2(1-\phi)^2 Y .$$

A sufficient condition for $q(X_{t+1})>0$ is that the discriminant of q be negative, which in turn is equivalent to

(51) $4(1-\eta)Y > \eta^2 W$.

Furthermore, (51) holds for all admissible η iff it holds for $\eta=\eta^{max}$. Noting that $\eta^{max}<1$ and $(1+\beta)\Delta>1$ implies that a sufficient condition for (51) is

(52) $4(1-\overline{\eta})Y > \overline{\eta} W$,

with $\overline{\eta} = (1-\phi)Y/(Y+\phi W)$. (52) is equivalent to (33).

Proof of Proposition 4

(a) $dR_t/d\eta<0$ can be shown by induction. First note that it is directly clear from (21) that $dR_{T-1}/d\eta<0$. For t=$T_1,...,T$-2,

$$R_t = (1-\eta)\frac{Y}{\phi\beta V}\left(1+\frac{W\phi}{Y\phi+\frac{(1-\phi)Y}{R_{t+1}}}\right) =: F(R_{t+1},\eta)$$

Since $F_1(R_{t+1}, \eta)>0$ and $F_2(R_{t+1}, \eta)<0$, the chain rule implies by induction that $dR_t/d\eta<0$. For $t=T_1-1$, we can show the following.

$$R_{T_1-1}=\tilde{J}_I(\hat{J}_D(R_{T_1+1}))=:G(R_{T_1+1},\eta),$$

with $G_1(R_{T_1+1}, \eta)>0$ and $G_2(R_{T_1+1}, \eta)=0$. Thus, $dR_{T_1+1}/d\eta<0$. For $t=1,...,T_1-2$, $R_t=J_I(R_{t+1})$, with $J_I'>0$, and the J_I function does not depend on η. Thus, by further induction $dR_t/d\eta<0$.

(b) For $t=1,..., T_1-1$, $Q_t=Q(R_t)$ with $Q'<0$ and $dR_t/d\eta<0$. Thus, $dQ_t/d\eta>0$. Further, $dQ_{T-1}/d\eta>0$ can be directly verified.

(c) For $t=1,..., T_1-1$, (35) implies that $d(\phi R_t+(1-\phi)Q_t)/dR_t<0$. Since $dR_t/d\eta<0$, the claim that $d(\phi R_t+(1-\phi)Q_t)/d\eta>0$ is established.

(d) From the proof of Proposition 2 we have that

$$p_1=\prod_{t=1}^{T_1-1}Q(R_t)R_{T_1}\frac{\eta}{1-\eta}\cdot c,$$

which has been shown to be increasing in η when (33) holds.

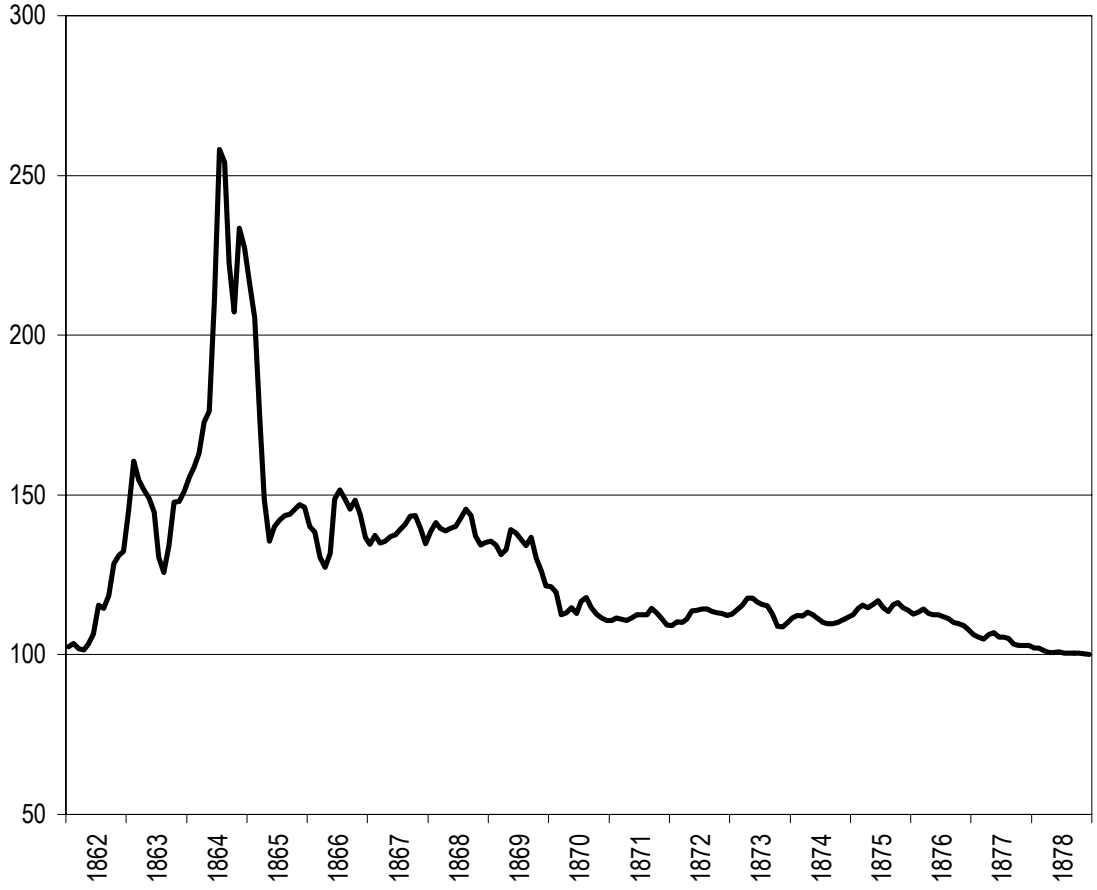
Table 1.
Backwards laws of motion of the real interest rate between $t=T$-1 and t=1, when gold reserves are accumulated.

t	γ_t	y_t	w_t	y_{t+1}	w_{t+1}	$J_t(R_t)$
T	0	$\hat{y}$	$\hat{w}$	y	w	
T-1	0	$\hat{y}$	$\hat{w}$	$\hat{y}$	$\hat{w}$	$\hat{J}_D(R_t)=\frac{\hat{Y}}{\phi\beta V}\left(1+\frac{\hat{W}\phi R_t}{\hat{Y}\phi R_t+(1-\phi)\hat{Y}}\right)$
$\vdots$	$\vdots$	$\hat{y}$	$\hat{w}$	$\hat{y}$	$\hat{w}$	
T_1+1	0	$\hat{y}$	$\hat{w}$	$\hat{y}$	$\hat{w}$	
T_1	γ	y	w	$\hat{y}$	$\hat{w}$	$\tilde{J}_I(R_t)=\frac{Y}{\phi\beta V}\left(1+\frac{W\phi R_t}{(Y-(1+\beta)\gamma)\phi R_t+(1-\phi)\hat{Y}}\right)$
T_1-1	γ	y	w	y	w	$J_I(R_t)=\frac{Y}{\phi\beta V}\left(1+\frac{W\phi R_t}{(Y-(1+\beta)\gamma)\phi R_t+(1-\phi)Y}\right)$
$\vdots$	$\vdots$	y	w	y	w	
2	γ	y	w	y	w	
1	γ	y	w	y	w	

Table 2.
Backwards laws of motion of the real interest rate between $t=T$-1 and t=1, when gold reserves are not accumulated.

t	γ_t	y_t	w_t	y_{t+1}	w_{t+1}	$J_t(R_t)$
T	0	$\hat{y}$	$\hat{w}$	y	w	
T-1	0	y	w	$\hat{y}$	$\hat{w}$	$\tilde{J}_D(R_t)=\frac{Y}{\phi\beta V}\left(1+\frac{\hat{W}\phi R_t}{Y\phi R_t+(1-\phi)\hat{Y}}\right)$
T-2	0	y	w	y	w	$J_D(R_t)=\frac{Y}{\phi\beta V}\left(1+\frac{W\phi R_t}{Y\phi R_t+(1-\phi)Y}\right)$
⋮	⋮	y	w	y	w	
T_1+1	0	y	w	y	w	
T_1	γ	y	w	y	w	$J_I(R_t)=\frac{Y}{\phi\beta V}\left(1+\frac{W\phi R_t}{(Y-(1+\beta)\gamma)\phi R_t+(1-\phi)Y}\right)$
⋮	⋮	y	w	y	w	
2	γ	y	w	y	w	
1	γ	y	w	y	w	

Figure 1.
The price of 100 gold dollars in terms of paper dollars during and after the U.S. Civil War



Source: Mitchell (1908/1966), Table 2, pp. 5-13.

Figure 2.
The backwards law of motion of the real interest rate (general form).

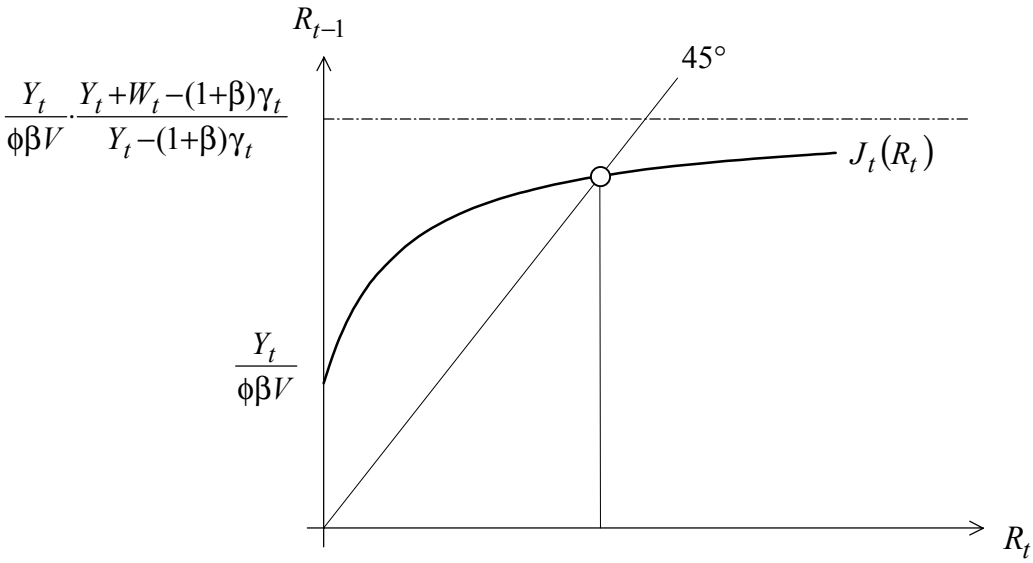


Figure 3.
Dynamic path of the real interest rate when gold reserves are accumulated.

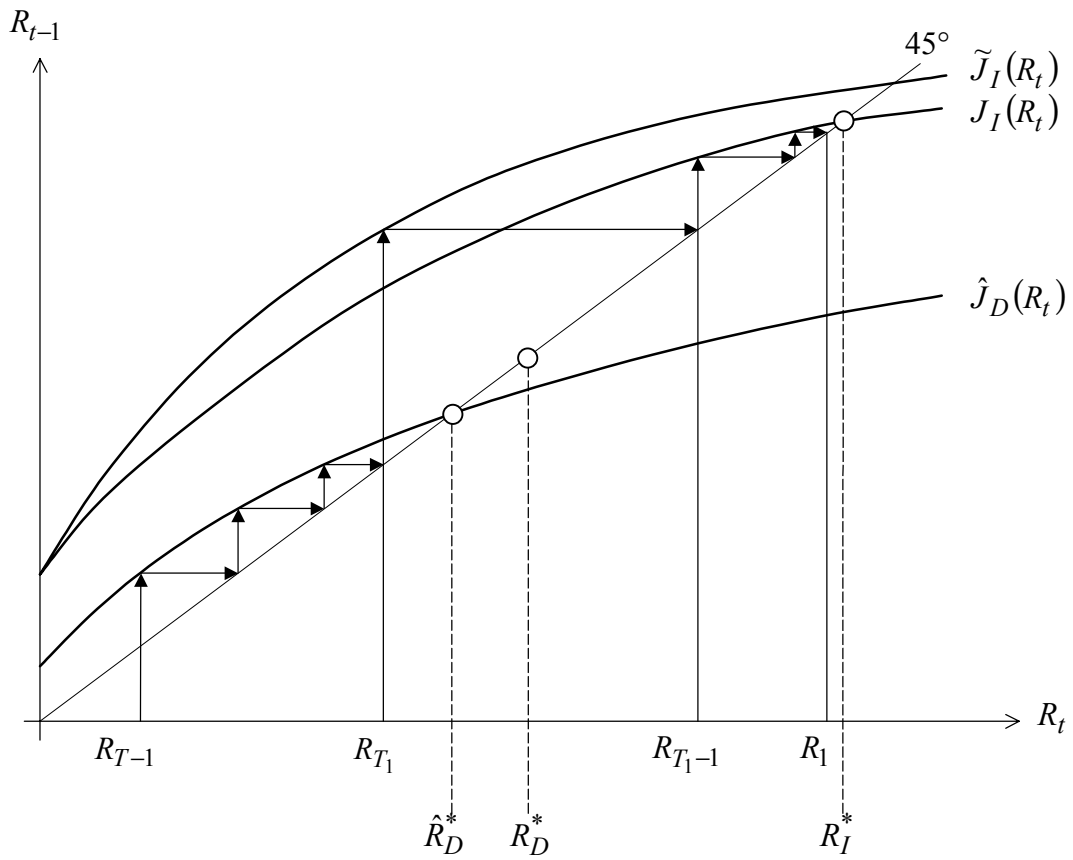
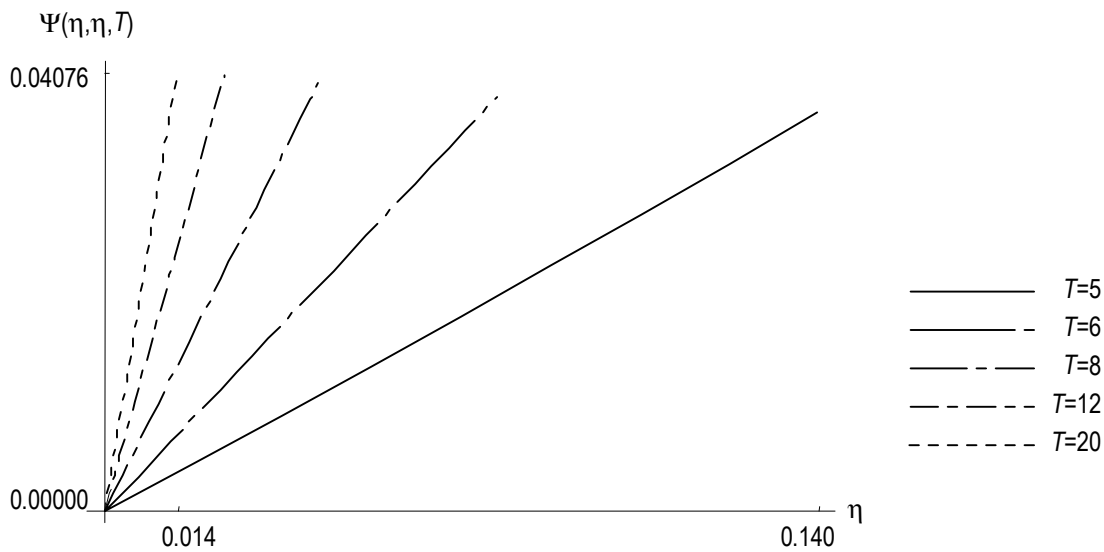


Figure 4.
Graph of the $\Psi(\eta,\eta,T)$ function when gold reserves are accumulated and taxes are proportional to agents' old age endowments.



Parameters: V=30, W=5, Y=65, ϕ=0.75, γ=2.4, β=0.9, δ=0.05, T_1=4.

Figure 5.
Dynamic properties of the price level paths as a function of policy parameters.

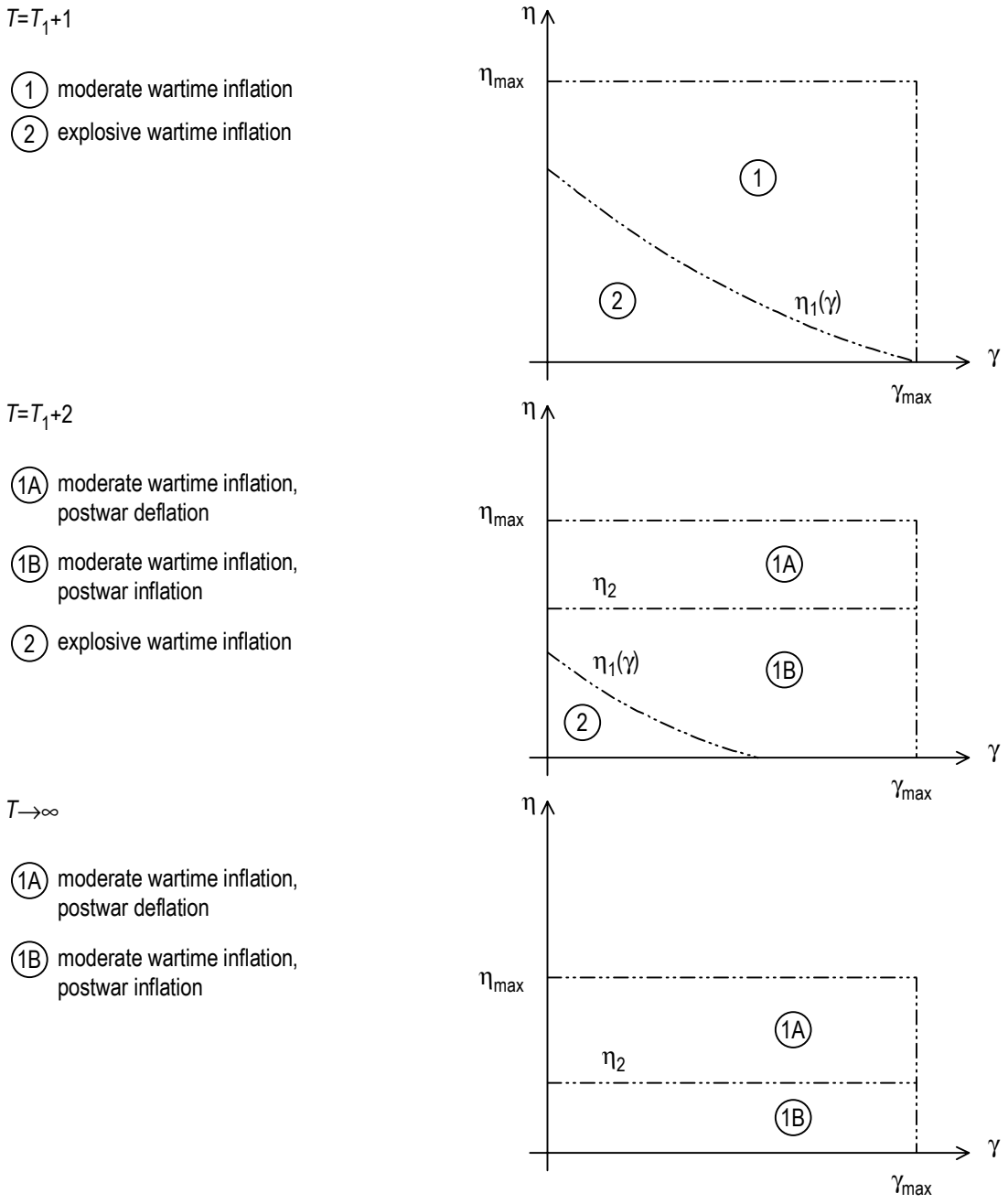
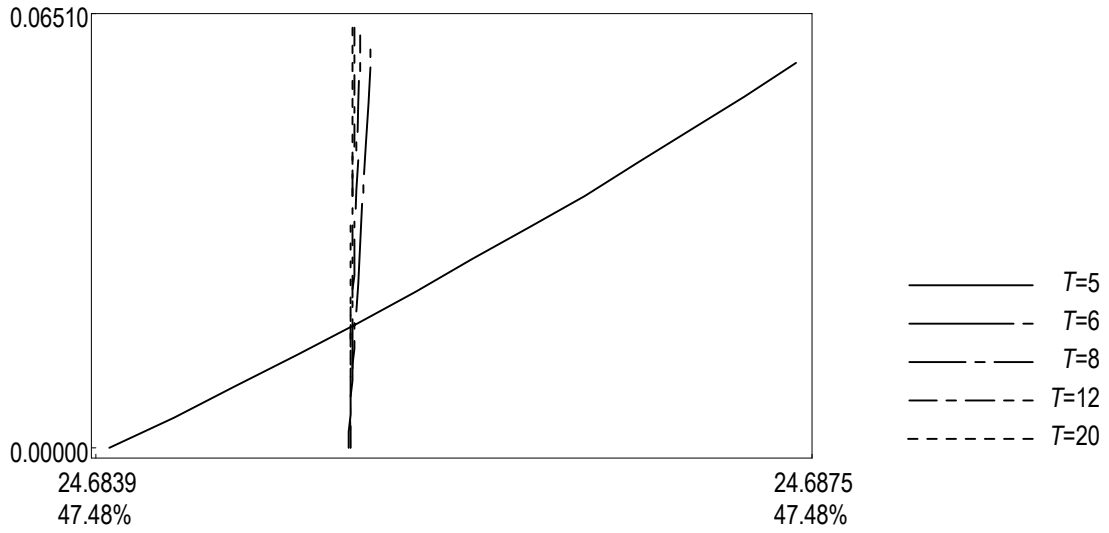
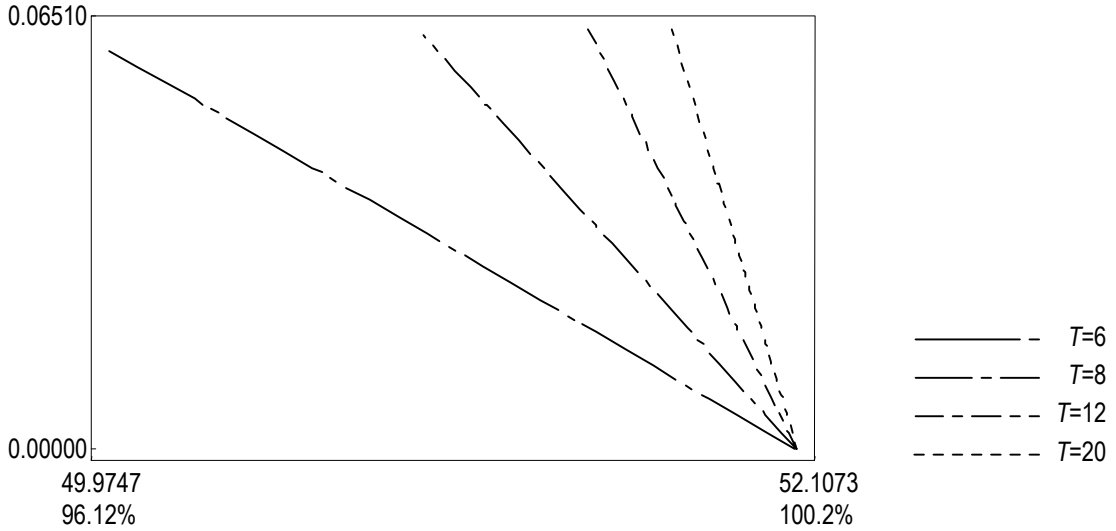


Figure 6.
Welfare of initial old and generations 1, …, T_1-1 (horizontal axis) as a function of $eb\varepsilon \cdot M_0$ (vertical axis).



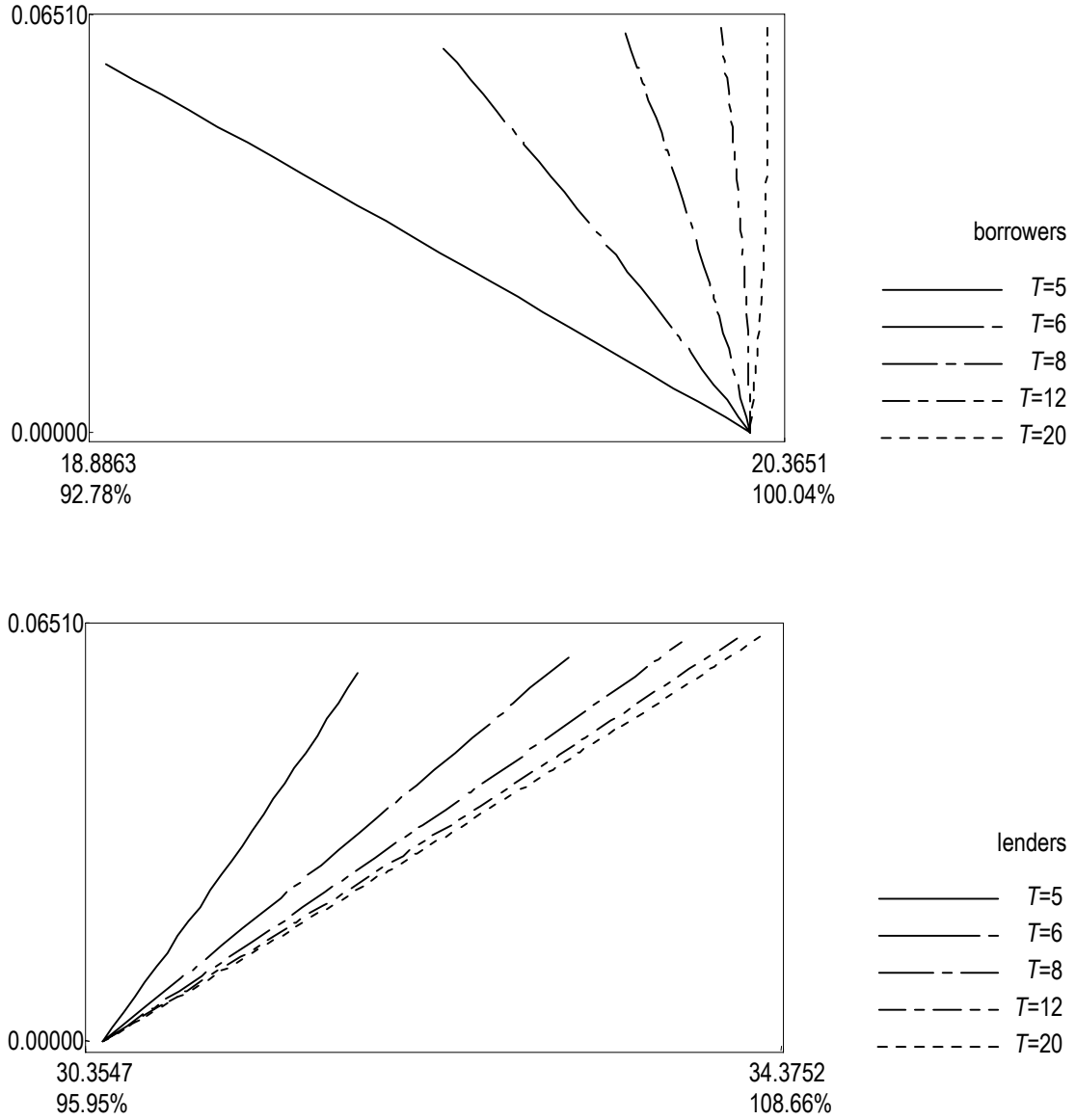
Parameters: V=30, W=5, Y=65, ϕ=0.75, γ=2.4, β=0.9, δ=0.05, T_1=4.
Note: Welfare as measured by (36), and as a fraction of post-stabilization steady state welfare.

Figure 7.
Welfare of generations $T_1, \ldots, T\text{-}2$ (horizontal axis) as a function of $eb\varepsilon \cdot M_0$ (vertical axis).



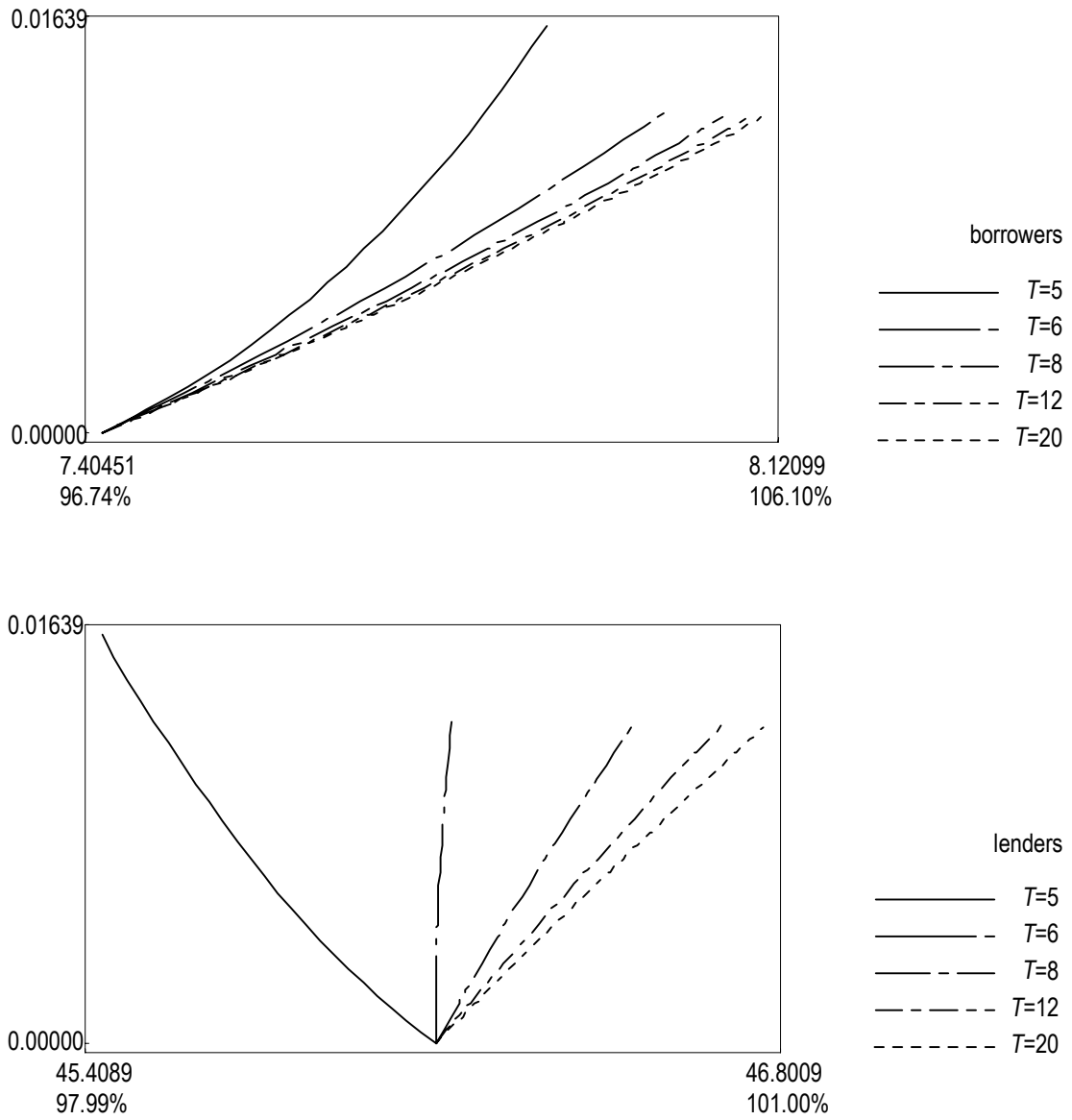
Parameters: V=30, W=5, Y=65, ϕ=0.75, γ=2.4, β=0.9, δ=0.05, T_1=4.
Note: Welfare as measured by (36), and as a fraction of post-stabilization steady state welfare.

Figure 8.
Welfare of generation T-1 borrowers and lenders (horizontal axis) as a function of $eb\varepsilon \cdot M_0$ (vertical axis), when aggregate wealth of borrowers is high.



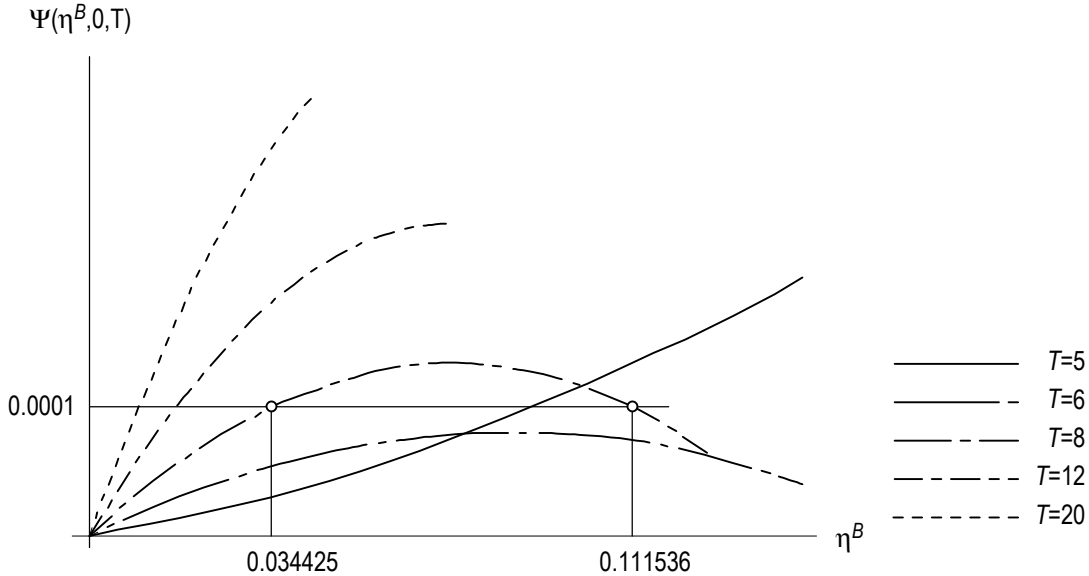
Parameters: V=30, W=5, Y=65, ϕ=0.75, γ=2.4, β=0.9, δ=0.05, T_1=4.
Note: Welfare as measured by (36), and as a fraction of post-stabilization steady state welfare.

Figure 9.
Welfare of generation T-1 borrowers and lenders (horizontal axis) as a function of $eb\varepsilon \cdot M_0$ (vertical axis), when aggregate wealth of lenders is high.



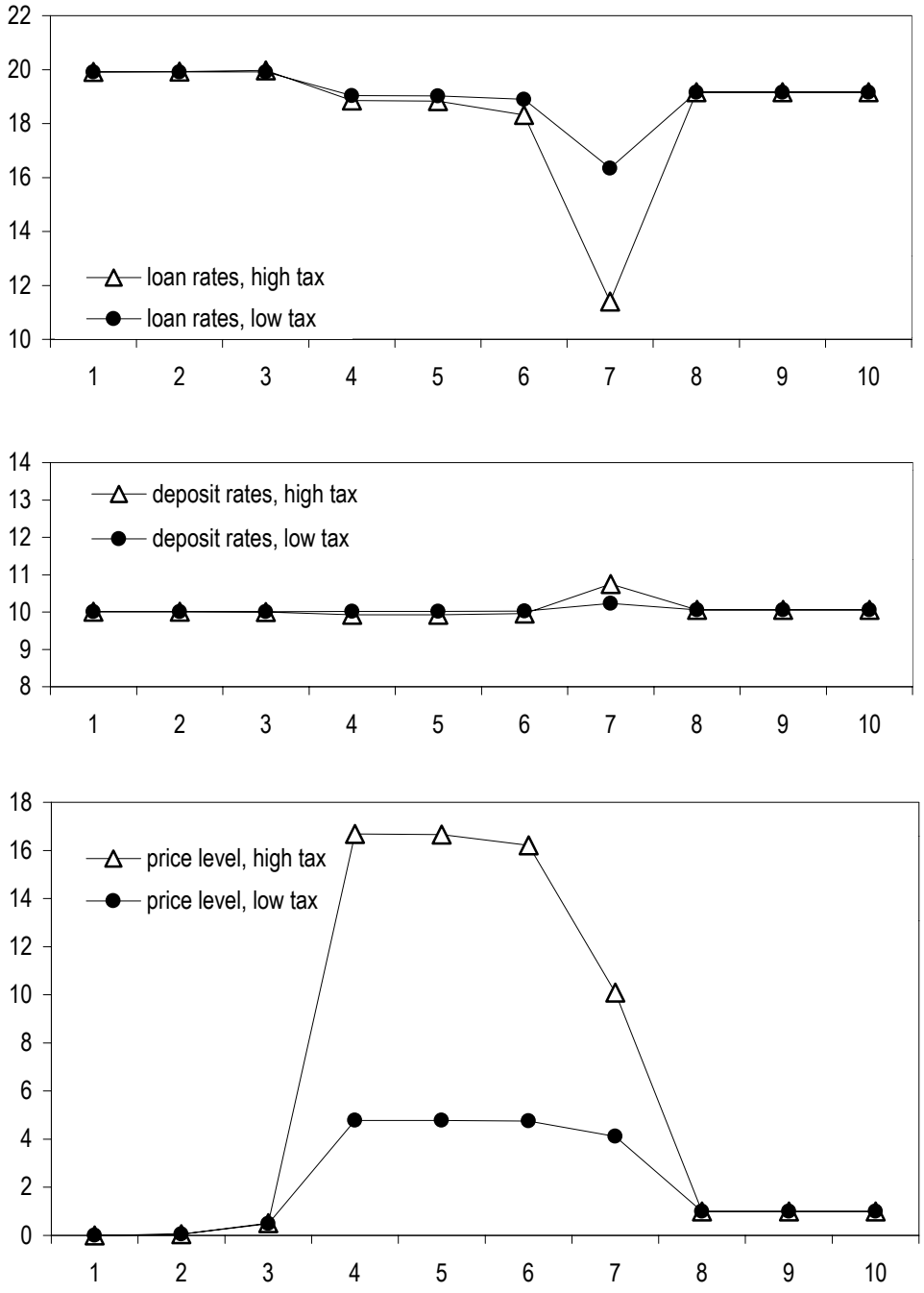
Parameters: V=30, W=45, Y=25, ϕ=0.75, γ=1.0, β=0.9, δ=0.05, T_1=4.
Note: Welfare as measured by (36), and as a fraction of post-stabilization steady state welfare.

Figure 10.
Example of non-monotonic $\Psi(\eta^B, 0, T)$ when only borrowers are taxed.



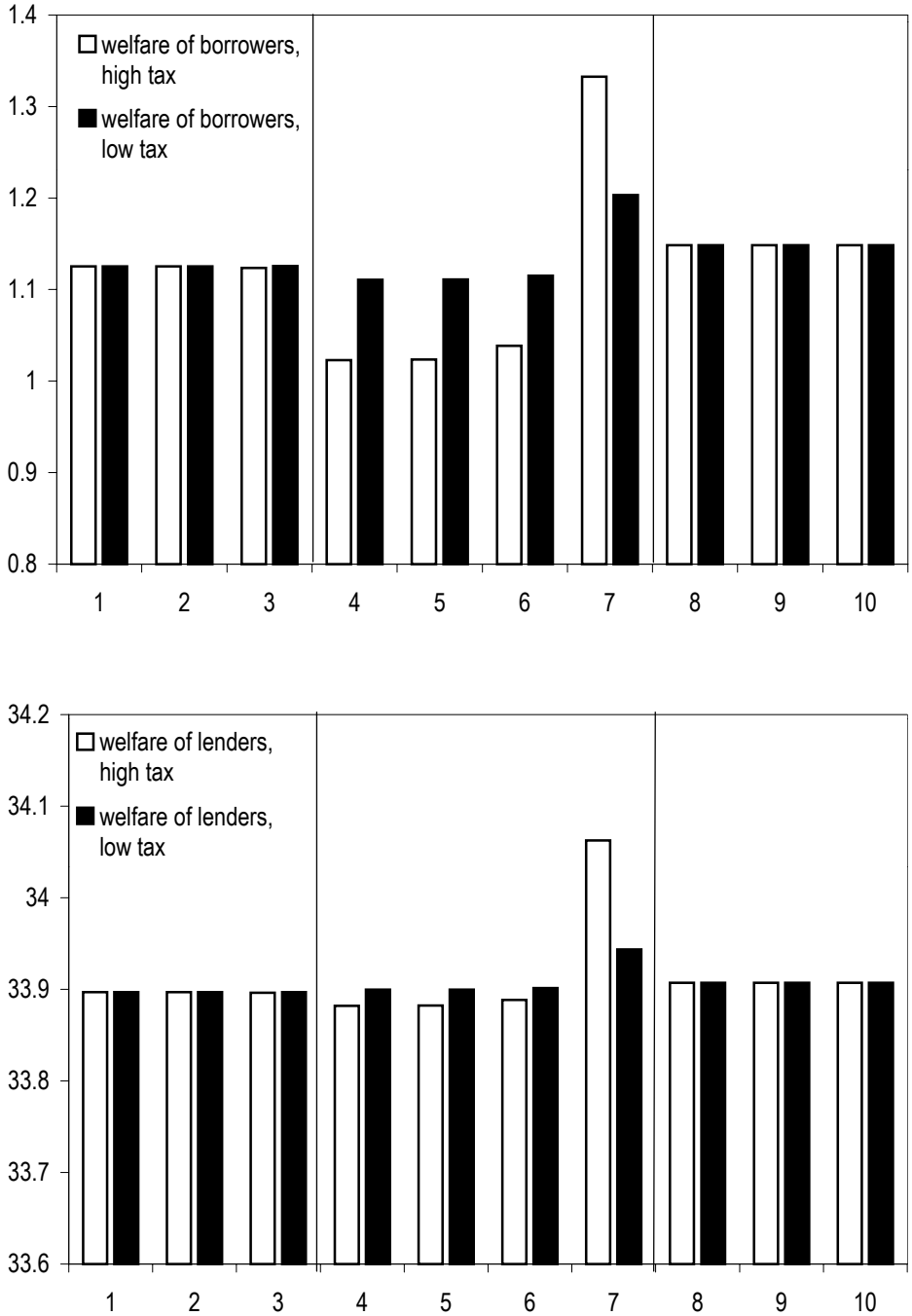
Parameters: V=10, W=80, Y=10, ϕ=0.5, γ=0.24, β=0.9, δ=0.05, T_1=4.

Figure 11.
The time path of loan rates, deposit rates and the price level under different taxes consistent with the same resumption date and parity.



Parameter values: T=8, low tax rate=3.4425%, high tax rate=11.1536%, and as in Figure 10.

Figure 12.
Welfare of borrowers and lenders under different taxes consistent with the same resumption date and parity.



Parameter values: T=8, low tax rate=3.4425%, high tax rate=11.1536%, and as in Figure 10.

Figure 13.
Dynamic path of the real interest rate when gold reserves are not accumulated.

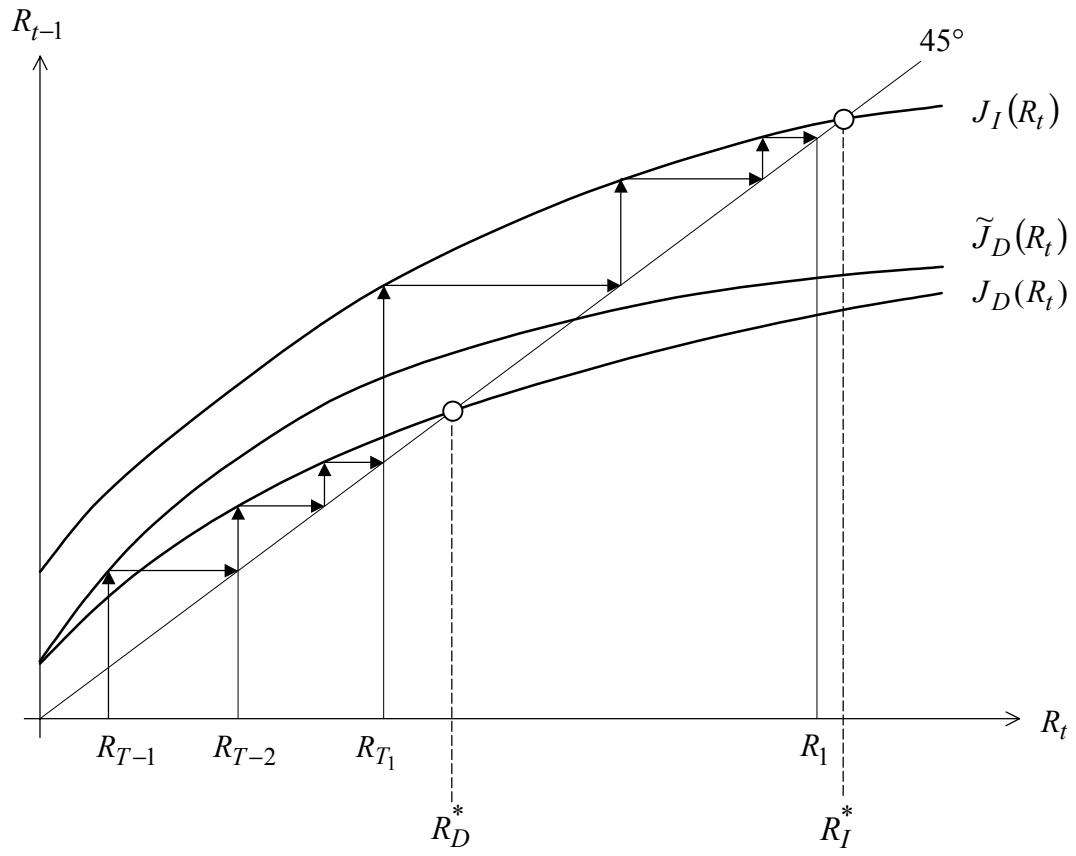
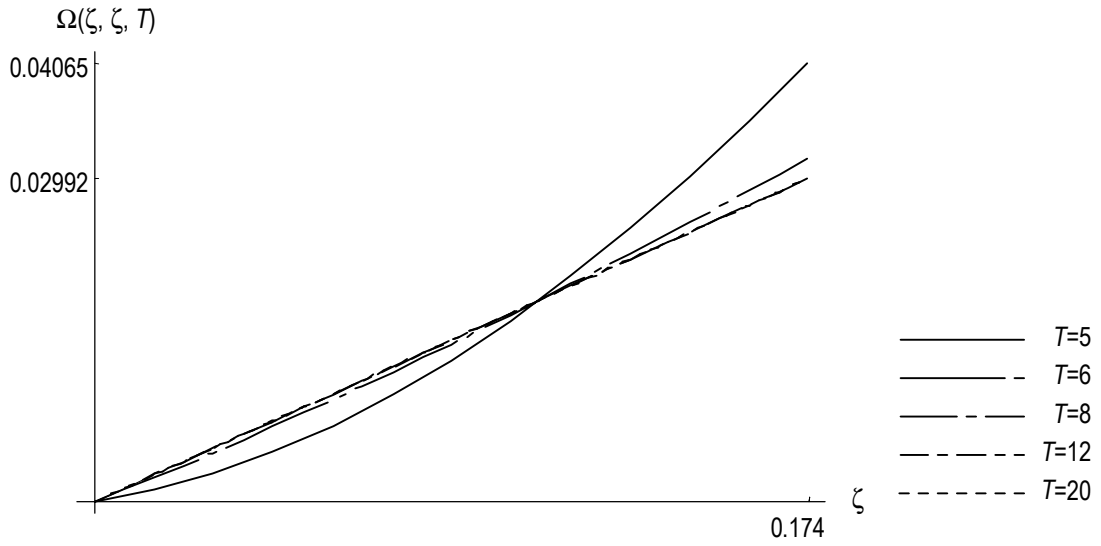


Figure 14.
Graph of the $\Omega(\zeta, \zeta, T)$ function when gold reserves are not accumulated.



Parameters: V=50, W=30, Y=20, ϕ=0.5, γ=4, β=0.9, δ=0.05, T_1=4.