Limited Commitment, Money and Credit*

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January 2000

Running Title: Limited Commitment, Money & Credit

Keywords: Money, credit, private information.

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*We thank Bruce Smith, Steve Williamson, and Randall Wright for their input on earlier drafts. The views stated herein are those of the authors and are not necessarily those of the Federal Reserve Bank of Cleveland or of the Board of Governors of the Federal Reserve System. All errors and shortcomings are ours.

Abstract:

This paper studies limited commitment and adverse selection in an economy in which private liabilities (inside money) can be used as instruments of intertemporal trade. The results suggest that in conjunction with adverse selection, the limited commitment problem may affect the behavior of intrinsically higher quality debtors more severely than lower quality ones. Nonetheless, a credit economy may function better when both problems are present than under limited commitment alone. Adding a fixed amount of money eases frictions associated with the credit market.

1 Introduction

This paper studies limited commitment and adverse selection in an economy in which private liabilities (inside money) can be used as instruments of intertemporal trade.[1] It compares equilibria in which the issuance of private liabilities suffers from both limited commitment and adverse selection with equilibria affected by limited commitment alone. It also examines the role that unbacked fiat currency (outside money) might play as a complement to imperfect credit markets.

This research is motivated by some questions regarding the feasibility of private credit instruments as media of exchange. First, to what extent do difficulties in enforcing debt contracts undermine the effectiveness of trading arrangements based on private credit? Second, how likely are markets to sort out debtors who differ in their intrinsic ability to redeem commitments? Third, does a freer flow of information in credit markets inevitably improve their functioning? Fourth, can outside money act as a complement to imperfect credit markets?[2] Such questions have been invoked both in historical debates about "free banking" (Selgin and White, 1994) and with respect to the emerging phenomenon of electronic cash (Schreft, 1997).

This paper addresses these questions in the context of a model in which both outside money and private credit may be used to facilitate trade. The model assumes two sets of infinite-lived agents who gain by repeatedly trading with each other. One set of agents receives one unit of a single, non-storable good in even periods while the second set receives

[1] The term limited commitment is closely related to moral hazard in that it covers opportunistic behavior arising from hidden actions or costly enforcement of contracts. In this paper, the term is used to describe the opportunistic repudiation of a debt contract because it is too costly for creditors to directly enforce repayment.

[2] Insights on these questions include: (1) Friedman (1960) discusses a number of problems that might arise with unregulated economies in which private issuers of fiduciary liabilities are tempted to renege on their obligations; (2) Hicks (1989) conjectures that high quality debtors are likely to prevail in competition with low quality ones since the latter are more likely to default out of the market; (3) Smith (1986) shows that private information about debtor quality can lead to non-existence of equilibrium; (4) Smith (1986) also shows that adding money to a credit economy based on private information about debtor quality can lead to *ex post* revelation of borrower type and can restore equilibrium over parameter values where such equilibrium did not exist.

one unit of a single, non-storable good in odd periods. In this environment non-circulating promissory notes (IOUs) of one-period maturity, issued by one set of agents and bought by the other, can be used as instruments of trade.

The set of agents who issue IOUs are sub-divided into two classes: 'safer' and 'riskier'. The former have a higher probability of being *able* to make good on their IOUs than the latter. If loan contracts were enforceable and creditors could distinguish between the two types of borrowers, private IOUs would be sufficient to realise all potential gains from trade; outside money would not play a complementary role in this case.[3] However, loan contracts are not strictly enforceable so that, ability to repay aside, both classes of borrowers may intentionally repudiate their IOUs. Production is observable, so that default due to illiquidity is forgiven, but intentional default is punished by excluding the offender from future trading opportunities.

The model is simple enough to allow for explicit solutions for at least the stationary equilibria. The results suggest that as long as credit markets are 'thick' *i.e.* enjoy a sufficiently large supply of funds in the steady state, intentional default is not observed in equilibrium regardless of whether or not creditors can distinguish between the two classes of borrowers.[4] This result is in the spirit of Williamson (1992), that regimes based on inside money may not display fraudulent behavior, even though the potential for fraud is present. As the supply of funds shrinks, agents begin to default with positive probability. In the case where borrower type is common knowledge, riskier borrowers are likely to engage in default before safer borrowers have an incentive to do so. With private information about borrower type, safer borrowers become more likely to engage in default. This suggests that in conjunction with a private information problem, the limited commitment problem may be more severe among intrinsically higher quality debtors than among their lower quality counterparts.

[3]Hence, although non-circulating debt cannot conventionally be considered as equivalent to inside money, in the present model, trades are simple enough that even non-circulating debt can achieve what conventional inside monies might achieve.

[4]A larger steady state supply of funds leads to higher gains from trade for borrowers and makes the penalty of being excluded from future borrowing opportunities more severe.

Nonetheless, in certain respects, a credit market performs worse under common knowledge than under private information. First, it might take a larger *ex ante* supply of funds to deter equilibrium default in the former case than in the latter one. Second, under common knowledge, the credit market may fail to deliver a non-trivial equilibrium of a type that could exist in an analogous situation under private information. This is in contrast with Smith (1986), where private information about debtor type leads to non-existence. These results suggest that in the presence of potentially opportunistic behavior, more information need not lead to better functioning credit markets.

Adding a fixed amount of money does indeed ease frictions associated with the credit market. As in Smith (1986), separating equilibria arise in which safer borrowers opt out of credit transactions entirely rather than face pooling with riskier borrowers. The set of such equilibria closely parallel equilibria that arise in a pure credit economy with common knowledge, with the exception that over parameter values for which the credit market becomes completely inactive, trades can still take place with all agents using the money market. Hence, adding money to a pure credit economy expands the set of active equilibria. However, the informational role of money is tenuous at least for some parameter values, over which a second type of equilibrium is possible in which safer borrowers use both money and credit while riskier ones use credit alone.

As discussed above, Smith (1986) is closely related to this paper. He shows that in an economy with private information about borrower type, equilibrium may cease to exist because of discontinuities in borrowers' budget sets associated with the decision on whether to reveal their type or not. In such an environment, it is shown that adding money can lead to revelation of borrower type and thereby to existence. Discontinuities also cause non-existence in our model, but are associated with the possibility of default and affect the demand for securities by lenders even when the underlying informational structure is based on common knowledge. Private information about the quality of debt instruments is also studied in Williamson (1992), who shows that legal restrictions on the issuance of inside

money can improve welfare by supporting an equilibrium in which fiat money is valued.

The problem of limited commitment in credit transactions has been previously studied in Bernhardt (1989). The concern is with the endogenous formation of trading relationships based on credit as opposed to demands for cash in advance. This is something taken for granted in our paper. At the same time, Bernhardt does not consider the interaction of limited commitment with adverse selection and how this affects the existence and nature of equilibrium, which is what this paper sets out to do. Other papers that have recently explored the coexistence of money and credit are Aiyagari, Wallace, and Wright (1996), Li (1998), Schreft (1992), and Shi (1995). These papers emphasize the role of frictions in the process of transactions and neither adverse selection nor limited commitment are considered.

The rest of the paper is as follows. Section 2 lays out the basic model. Section 3 examines equilibria in a pure credit economy. Section 4 allows for the coexistence of credit and fiat currency. Section 5 concludes.

2 Model.

Time is discrete, lasts forever, and is indexed by $t = 0, 1, 2, \cdots,$. Each period is associated with one of two seasons, "Fall" and "Summer", which alternate cyclically. The index is normalized so that fall arrives in even-numbered periods. There are two non-storable consumption goods that are produced in different seasons. To help fix ideas for what follows, imagine the two goods as Apples and Bananas, with apples produced in fall and bananas in the summer.

There is a continuum of infinitely-lived agents and a countably infinite number of locations or islands. Each agent is randomly assigned to an island at the start of each fall, residing there for two seasons.[5] Once all agents have arrived at their locations, communication

[5]What is important is that agents stay on a given island for a finite number of seasons, which exceeds unity. A one-season stay would prevent loan contracts from arising. At the other extreme, assuming that agents stay on the same island infinitely might negate the role of idiosyncratic risk, described below, which is important for our results.

between locations ceases for the remainder of their stay. At the end of two periods all agents are randomly reassigned to new islands.

Since relocation occurs in the fall, apple production precedes banana production. Each agent is endowed with an exogenous ability to produce either apples or bananas, but not both. Thus, apple producers will be referred to as type A and banana producers as type B. Due to the large numbers of agents and islands and the randomness of assignments, the respective measures of type A and type B agents coexisting on a given island is the same across islands and over time, reflecting the relative measure of the two types in the population at large. N_A and N_B denote the measures of types A and B per island, respectively.

Each producer can produce costlessly a fixed amount of their particular production good, normalized to unity. A greater amount entails infinite costs. Production can be risky, however. For our purposes, no generality is lost in assuming that apple production is riskless, but banana producers face the risk of crop destruction through pestilence. Moreover, one sub-type of banana producers always faces greater risk than another sub-type, regardless of the locations visited. Let π^S be the probability that the crop of a safer banana producer is successful in any given season, and π^R the probability that the crop of a riskier banana producer is successful in any given season. Unsuccesful producers lose their entire crop. Assume that:

Assumption 1 $0 < \pi^R < \pi^S < 1$.

The fraction of safer banana producers is σ and the fraction of riskier ones is $1 - \sigma$. The aggregate output of apples is N_A per island, while the aggregate output of bananas is ηN_B, where $\eta \equiv \pi^S \sigma + \pi^R(1 - \sigma)$. Let ψ denote the ratio $N_A/\eta N_B$. This ratio will be used to characterize the set of equilibria.

To motivate exchange, assume that agents have a relative preference to consume the good they do not produce: apple producers have a relative preference for consuming bananas while banana producers have a relative preference for consuming apples. Agents are risk-neutral

and have preferences:

$$\textbf{Type A:} \quad U^{A,h}(0) = \Sigma_{t=0}^{\infty}\beta^t \left[\alpha c_A^h(t) + c_B^h(t)\right]$$

$$\textbf{Type B:} \quad U^{B,k}(0) = \Sigma_{t=0}^{\infty}\beta^t \left[c_A^k(t) + \alpha c_B^k(t)\right],$$

where $U^{A,h}(0)$ is the lifetime utility level for the hth type-A agent at time $t = 0$; $U^{B,k}(0)$ is the counterpart for the kth type-B agent. Similarly, $c_A^i(t)$, $c_B^i(t)$ denote allocations of goods A and B at time t for the ith agent of either type. α is a parameter reflecting relative preferences over the two goods; $\alpha < 1$ is necessary for mutual gains from trade.

Due to the seasonal nature of production and the non-storability of goods, simultaneous exchange is not possible. Thus, for any agent i, regardless of type, $c_B^i(t) = 0$ for t even, and $c_A^i(t) = 0$ for t odd. The opportunity remains, however, for agents to trade intertemporally: apple producers could give their output to banana producers in the fall and receive the latter's output in the summer. The following assumption ensures gains from intertemporal trade:

Assumption 2 $\quad 0 \leq \alpha < \beta \leq 1.$

A government exists with jurisdiction over all islands. Its functions include issuing unbacked fiat currency and exercising legal control over agents (as detailed in the following two sections).

In this economy, both money and short-term credit may be used to facilitate trade. Two alternative arrangements are examined:

(I) An economy in which all trades take place through credit. Securities of one-period maturity are sold by type B agents to type A agents at the start of every fall season and redeemed in the following summer season, before all agents disperse to their new locations.

(II) An economy in which a single fiat currency is issued by the government and, if valued by private agents, may be used alongside private securities to facilitate trade at all locations. Unlike securities, money may be traded across locations: agents who end up holding money at the end of their stay on one island may use it to purchase goods on the next island.

3 Credit Economy.

In this section, it is assumed that markets exist in which apple producers buy securities of one-period maturity from banana producers. The former will be referred to as "lenders" and the latter as "borrowers". A security issued at time t promises, conditional upon having a good, to pay a fixed amount at $t + 1$.

A lender faces two kinds of repayment risk: *liquidity* risk, arising from the failure of the borrower to receive his output; and *strategic* risk, arising from intentional non-repayment despite successful production of output. The former risk is exogenous, while the latter is endogenous. The term "default", when used without qualification, will be understood in this paper to refer to non-repayment of debt arising from strategic, rather than liquidity, reasons.

If default were to go unpunished, borrowers would engage in it at every opportunity, making credit transactions impossible. A successful crop is assumed to be observable so that creditors can distinguish between intentional and liquidity default. At the same time, neither creditors nor the government have the power to confisicate output. Hence, repayment cannot be strictly enforced. However, should an act of intentional default take place, creditors can credibly report it to the government, which has the power to punish offenders. The threat of *ex post* punishment serves as an *ex ante* device which can make credit transactions viable.[6]

What is required for the results of this paper is that liquidity default is treated differently from, and punished less severely than, intentional default, and that punishment for the latter is finite and contains a fixed element. An example of such punishment would be to exclude the offender from future transactions for a fixed minimum period which is independent of the size of the loan defaulted on, plus a possible variable period which depends on the magnitude of default. Since a variable penalty by itself would play no role in the analysis it

[6] An alternative to government enforcement would be to assume that creditors who suffer an intentional default can "label" the offender, identifying them to creditors met in subsequent locations. These creditors would not lend to a labeled borrower. While this reputational enforcement mechanism would suffice to render credit transactions feasible in a pure credit economy, it might not do so in a combined monetary-credit economy. This is why the stronger mechanism of government enforcement is chosen.

is ignored; while, for the sake of analytical tractability, the fixed penalty is assumed to involve exclusion from market transactions for all future periods.[7] By the same token, punishment for liquidity default is ignored, mainly because there is little incentive for lenders to impose such punishment in the present model where there is no hidden information about outputs.

Equilibrium in competitive markets requires the *ex ante* return on securities issued by safer and riskier borrowers to be equal. Since the two types of borrowers face different production probabilities and are likely to behave differently with respect to strategic default, each type of security will sell at a separate discount when lenders can differentiate between the two types of borrowers. This case is analyzed first, while the case where different types of borrowers cannot be distinguished is analyzed next.

3.1 Common Knowledge.

The output probability of each borrower is common knowledge. While part of the probability of non-repayment is endogenous and consists of intentional default, creditors can assess this probability in equilibrium. Let $\lambda^i(t+1)$, $i = \{S, R\}$, $t = 0, 2, \cdots$, denote the probability that a borrower of risk type i will default at time $t+1$ on a security sold at time t. Let $q^i(t+1)$ denote the repayment probability of securities by subtype i. Then

$$q^i(t+1) = \left(1 - \lambda^i(t+1)\right) \pi^i \tag{1}$$

Let $\theta^i_A(t)$, $i = S, R$ denote the quantity of securities of each risk type bought by a lender at time t, $t = 0, 2, \cdots$. Let $p^i(t)$ denote the price of each security. $p^i(t)$ measures the amount of time t goods that are given up in exchange for one expected unit of time $t+1$ goods. Each

[7] Exclusion from all future transactions entails a finite penalty in that the offender can still consume their own output. A finite penalty is consistent with the principle of limited liability, fundamental to much of contract law. Further, some equilibria of interest in the model involve default by at least some agents. This would not be possible under an infinite penalty.

apple producer/lender maximizes a sequence of two-period problems,

$$
\begin{aligned}
V_A(t) = \max_{\theta_A^S(t),\theta_A^R(t)} & \Bigg\{ \alpha(1-\theta_A^S(t)-\theta_A^R(t)) \\
& + \beta\left(q^S(t+1)\frac{\theta_A^S(t)}{p^S(t)} + q^R(t+1)\frac{\theta_A^R(t)}{p^R(t)}\right) + \beta^2 V_A(t+2)\Bigg\} \\
s.t.: \quad & 1 \geq p^S(t)\theta_A^S(t) + p^R(t)\theta_A^R(t)
\end{aligned} \tag{2}
$$

for each $t = 0, 2, \cdots$. The solution to (2) satisfies:

$$
\frac{q^S(t+1)}{p^S(t)} = \frac{q^R(t+1)}{p^R(t)}; \tag{3}
$$

$$
\frac{q^S(t+1)}{p^S(t)} \begin{cases} > \\ = \\ < \end{cases} \frac{\alpha}{\beta} \implies \theta_A(t) = \begin{cases} 1 \\ \in [0,1] \\ 0. \end{cases} \tag{4}
$$

For banana producers/borrowers, the problem involves two choices. Provided that they have not been in default up to time t, they decide $\theta_B^i(t+1)$, the fraction of their expected $t+1$ output to borrow against at time t, and $\lambda^i(t+1)$, the probability with which they intentionally default on time t debt, $i = \{S, R\}$, $t = 0, 2 \cdots$.

$$
\begin{aligned}
V_B^i(t) = \max_{\theta_B^i(t+1),\lambda^i(t+1)} & \Bigg\{ p^i(t)\theta_B^i(t+1) \\
& + \ \beta\pi^i\left(\lambda^i(t+1)\tilde{V}_B^i(t+1) + (1-\lambda^i(t+1))(\alpha(1-\theta_B^i(t+1)) + \beta V_B^i(t+2))\right) \\
& + \ (1-\pi^i)\beta^2 V_B^i(t+2)\Bigg\} \\
s.t.: \quad & \theta_B^i(t+1), \lambda^i(t+1) \in [0,1]
\end{aligned} \tag{5}
$$

where $\tilde{V}_B^i(t+1)$ denotes the payoff from defaulting at $t+1$.

$$
\tilde{V}_B^i(t+1) = \alpha + \frac{\pi^i\beta^2\alpha}{1-\beta^2};
$$

i.e. the defaulter gets additional consumption at one point in time and a stationary path of expected consumption thereafter.

Hence,

$$p^i(t) \begin{cases} > \\ = \\ < \end{cases} \pi^i \beta \alpha (1 - \lambda^i(t+1)) \quad \Longrightarrow \quad \theta_B^i(t+1) = \begin{cases} 1 \\ \in [0,1] \\ 0 \end{cases} \tag{6}$$

$$\beta V_B^i(t+2) \begin{cases} > \\ = \\ < \end{cases} \tilde{V}_B^i(t+1) - \alpha(1 - \theta_B^i(t+1)) \quad \Longrightarrow \quad \lambda^i(t+1) = \begin{cases} 0 \\ \in [0,1] \\ = 1 \end{cases} \tag{7}$$

In order to describe market clearing, let the fraction of borrowers of sub-type i, $i = \{S, R\}$, who are eligible to borrow at time t be denoted by $\phi^i(t)$. $\phi^i(t)$ depends on past defaults

$$\phi^i(t) = (1 - \pi^i \lambda^i(t-1))\phi^i(t-2). \tag{8}$$

As a predetermined variable, $\phi^i(0)$ can take on any value between 0 and 1. However, without loss of generality, $\phi^i(0) = 1$, $i = \{S, R\}$ is assumed throughout the paper.

The market clearing conditions are

$$N_A \theta_A^S(t) = \sigma N_B \phi^S(t) p^S(t) \theta_B^S(t+1) \tag{9}$$

$$N_A \theta_A^R(t) = (1-\sigma) N_B \phi^R(t) p^R(t) \theta_B^R(t+1) \tag{10}$$

for $t = 0, 2, \cdots$.

Definition 1 *An equilibrium in the credit economy with common knowledge is described by a sequence of security prices $\{p^i(2t)\}_{t=0}^{\infty}$, repayment probabilities $\{q^i(2t+1)\}_{t=0}^{\infty}$, lending decisions $\{\theta_A(2t)\}_{t=0}^{\infty}$, borrowing decisions $\{\theta_B^i(2t+1)\}_{t=0}^{\infty}$, decisions to default $\{\lambda^i(2t+1)\}_{t=0}^{\infty}$ and fractions of eligible borrowers $\{\phi^i(2t)\}_{t=0}^{\infty}$, $i = (S, R)$ that satisfy equations (3), (4), (6)-(10) at each point of time.*

Definition 2 *A time-t stationary equilibrium in the credit economy is an equilibrium in which stationary sequences of prices, p^i, repayment probabilities, q^i, lending decisions, θ_A, borrowing decisions, θ_B^i, and fractions of eligible borrowers, ϕ^i, $i = \{S, R\}$ exist from some date t.*

Definition 3 *A default-free stationary equilibrium is a stationary equilibrium from $t = 0$.*

In a time-t stationary equilibrium, the rate of intentional default cannot be positive after time $t+1$. In a default-free stationary equilibrium, the rate of intentional default has to be zero at $t = 1$ and in all subsequent periods; hence $\phi^i = 1$, $i = \{S, R\}$ for all periods.

The following lemmas establish some results about the borrowing and strategic default behavior of the two types of borrowers.

Lemma 1 *Given a stationary price p^i of securities issued by borrowers of risk type i, $i = \{S, R\}$, a borrower is induced not to default if and only if*

$$p^i \geq \frac{\alpha}{\beta} - \alpha\beta\left(1 - \pi^i\right) \equiv \hat{p}^i$$

Proof: From equation (7), $\beta V_B^i \geq \alpha\theta_B^i + \pi^i\beta^2\alpha/(1-\beta^2)$ implies that either $\lambda^i = 0$ or $\beta V_B^i = \alpha\theta_B^i + \pi^i\beta^2\alpha/(1-\beta^2)$. In either case,

$$V_B^i = p^i\theta_B^i + \beta\pi^i\alpha(1-\theta_B^i) + \beta^2 V_B^i.$$

Hence,

$$V_B^i = \frac{(p^i - \beta\pi^i\alpha)\theta_B^i}{1-\beta^2} + \frac{\beta\pi^i\alpha}{1-\beta^2} \geq \frac{\alpha\theta_B^i}{\beta} + \frac{\beta\pi^i\alpha}{1-\beta^2}$$

and therefore,

$$\frac{p^i - \beta\pi^i\alpha}{1-\beta^2} \geq \frac{\alpha}{\beta}$$

which, after rearranging, yields the desired inequality. ■

Lemma 2 *Given a sequence of security prices, $\{p^i(t+2s)\}_{s=0}^{\infty}$, $p^i(t+2s) < (>) \hat{p}^i$ for some s and $p^i(t+2s) \leq (\geq) \hat{p}^i$ for all other s implies that $\lambda^i(t-1) = 1$ $(= 0)$.*

Proof: Follows from equation (7) and Lemma 1. ■

Lemma 3 *Given a stationary price, p^i of securities i, $p^i \geq \hat{p}^i$, $\theta_B^i = 1$, $(i = S, R)$.*

Proof: Follows from equation (6), since

$$\begin{aligned}\hat{p}^i &\equiv \frac{\alpha}{\beta} - \alpha\beta\left(1 - \pi^i\right) \\ &= \pi^i\alpha\beta + \alpha\left[\frac{1}{\beta} - \beta\right] \\ &> \pi^i\alpha\beta(1 - \lambda^i)\ \forall\ \lambda^i \in [0, 1]\end{aligned}$$

■

Lemmas 1 and 2 suggest that if borrowers repeatedly receive at least a critical price for their securities, they will be marginally induced not to default. Lemma 3 states that if the price of securities is high enough to deter certain default, it is also high enough to make borrowers want to participate fully in exchange.

One issue that arises is what happens if borrowers face a zero price at some time. Some of our analysis in this section examines the possibility of an equilibrium in which the price of risky securities starts at $p^R(0) = 0$ and then becomes positive at a later date. The following lemma pertains to such a case. It is stated for either type, but the implications will be explicitly studied for risky borrowers.

Lemma 4 *Given a sequence of security prices, $\{p^i(t+2s)\}_{s=0}^{\infty}$, if $p^i(t) = 0$ and $p^i(t+2s) \geq 0$ for $s \geq 1$, with a strict inequality for at least some s, then $\theta_B^i(t+1) = 0$.*

Proof: Suppose the lemma is not true, so that $p^i(t) = 0$, $p^i(t+2s) > 0$ for some $s \geq 1$, but that $\theta_B^i(t+1) > 0$. There are two possibilities. If the borrower intends to repay the loan with certainty, his payoff at t is:

$$V_B^i(t)\big|_{\theta_B^i(t+1)>0} = 0 + \beta\pi^i\alpha(1 - \theta_B^i(t+1)) + \beta^2 V^i B(t+2)$$

which is clearly decreasing in $\theta_B^i(t+1)$. If the borrower intends to repay with probability less than one, then his payoff is the same as if he intends to strategically default with probability one.

$$V_B^i(t)\big|_{\theta_B^i(t+1)>0} = 0 + \beta\pi^i \left[\alpha + \frac{\beta^2\pi\alpha}{+}\beta^2(1-\pi^i)V_B^i(t+2)\right].$$

Choosing $\theta_B^i(t) = 0$, his utility would be:

$$V_B^i(t)\big|_{\theta_B^i(t+1)=0} = \beta\pi^i\alpha + \beta^2 V_B^i(t+2)$$

where the strategy of consuming autarkically from $t = 2$ until $t + 2s$, then borrowing at the positive price $p^i(t + 2s)$ followed by a certain default, yields a lower bound on $V_B^i(t + 2)$

$$V_B^i(t+2) \geq \frac{\beta\pi^i\alpha}{1-\beta^2} + \beta^{2s-3}p^i(t+2s)$$

where $V^iB(t + 2)$ is defined over the simple strategy of consuming autarkically.

Hence $V_B^i(t + 2) > \beta\pi^i\alpha/(1 - \beta^2)$. But this means that for any $\pi^i > 0$, $V_B^i(t, 0) > V_B^i(t, \theta_B^i(t + 1))$. ■

Intuitively, Lemma 4 states that so long as the ability to borrow has some positive future value, a borrower will want a strictly positive price for securities today. This behavior is induced by the fact that the penalty for default is independent of the size of the default. If it were not so, then borrowers might be willing to engage in trades at arbitrarily small prices, so long as that meant incurring a commensurately small penalty.

We now compare the behavior of the two sub-classes of borrowers. Note that $\hat{p}^S > \hat{p}^R$, safer borrowers require a higher critical price for their securities to induce repayment. This is because they have less to lose from being punished should they default. At the same time, safer borrowers *receive* a higher price than riskier ones. This premium is high enough to make safer borrowers less likely to default. Define

$$\tilde{p}^i = \frac{\hat{p}^i}{\pi^i} = \frac{1}{\pi^i}\left[\frac{\alpha}{\beta} - \alpha\beta\right] + \alpha\beta$$

Since $\tilde{p}^i$ decreases in π^i, $\tilde{p}^R > \tilde{p}^S$. This suggests that riskier borrowers are more likely to default in equilibrium

Lemma 5 *Given stationary prices p^i of securities issued by borrowers of risk type i, $i = \{S, R\}$, if $p^R \geq \hat{p}^R$, then $p^S > \hat{p}^S$.*

Proof: For lenders to be indifferent between securities, from equation (3)

$$p^S = \frac{q^S}{q^R} p^R$$

Now suppose that $p^R \geq \hat{p}^R$, so that, from Lemma 1, $q^R = \pi^R$. Also, conjecture that $q^S = \pi^S$. This implies the following series of inequalities arises

$$p^S \geq \pi^S \frac{\hat{p}^R}{\pi^S} \geq \pi^S \tilde{p^R} > \pi^S \tilde{p}^S > \hat{p}^S$$

which, in turn, verifies the conjecture that $q^S = \pi^S$ on the basis of Lemma 1. ■

Stationary equilibria can be characterized as functions of a single parameter, ψ, defined above as the ratio of outputs, $N_A/\eta N_B$. The characterization is simplified if we assume:

Assumption 3 $\pi^R > \frac{\alpha^2(1-\beta^2)}{\beta^2(1-\alpha^2)}$

Assumption 3 implies that $\beta/\alpha > \tilde{p}^R$. Note that the qualitative results do not depend on this property.[8]

Rather than provide an exhaustive list of possibilities associated with possible values of ψ, we shall restrict our discussion to some cases which establish the following points: (i) despite the potential for intentional default, in equilibrium none might take place, (ii) an economy based on credit can cope with a limited amount of equilibrium default, (iii) if the limited commitment problem associated with one set of borrowers is severe enough, but not too severe, a stationary equilibrium may cease to exist.

Note that ψ is a measure of the per-capita supply of funds facing borrowers. Higher ψ implies more potentially more favorable terms of trade for them. With this in mind, define the following threshold values: $\psi_1 = \tilde{p}^R$; $\psi_2 = \left((1 - (\pi^R)^2(1-\sigma)/\eta\right)\tilde{p}^R$; $\psi_3 = \pi^S \sigma \tilde{p}^R/\eta$. It is easy to show that $\psi_1 \geq \psi_2 \geq \psi_3$.

[8] It merely ensures that default-free stationary equilibria exist in which $\theta_A^S + \theta_A^R = 1$. This in turn simplifies the characterization of equilibria in which default *does* take place.

An interval corresponding to high values of ψ; in particular $\psi \geq \psi_1$, leads to default-free stationary equilibria. This because at high values of ψ, both types of borrowers can receive high enough security prices to repay honestly. Proposition F.1 in the Appendix proves this for $\psi \in [\psi_1, \beta/\alpha]$.[9]

For $\psi < \psi_1$, the terms of trade facing riskier borrowers can no longer suffice to altogether prevent default. Nonetheless, equilibria can be constructed in which a unique fraction of riskier borrowers default and get excluded from receiving credit at $t = 1$. This allows the terms of trade facing the remaining fraction of riskier borrowers to rise sufficiently to deter any subsequent strategic defaults. Hence, a time-2 stationary equilibrium arises, associated with a fraction of riskier borrwers which remains stable from $t = 2$ onwards. Existence of this type of equilibrium is proven in Proposition F.2 in the Appendix.

Close to ψ_2, the price of riskier securities required to satisfy lenders' incentives can become so small, that riskier borrowers do not issue any at $t = 0$. This can lead to non-existence of equilibrium. In order to motivate this possibility, note that self-fulfilling beliefs on the part of lenders can lead at all parameter values to trivial equilibria in which one or both types of securities never get traded, *i.e.* consistent with the beliefs of lenders, an equilibrium exists in which $\lambda^i(t+1) = 1$, $p^i(t) = 0$, $i = S, R$, and no trade in the ith-type securities takes place. If a lender does deviate once and offers a positive price to borrowers, this results in certain default. Having acknowledged this possibility, we focus on equilibria in which lenders' beliefs are derived from fundamental parameters of the economy.

Proposition 1 *For $\psi \in (\psi_3, \psi_2]$, there is no non-trivial stationary equilibrium,* i.e. *there is no time-t stationary equilibrium in which riskier securities get traded, nor one in which only safer securities get traded, nor is autarky an equilibrium.*

The reader is referred to the Appendix for a proof. The non-existence result pertains to a particular class of equilibria, *i.e.* equilibria that achieve stationarity over time. As noted

[9]The case of $\psi > \beta/\alpha$ is qualitatively analogous but involves partial trade by lenders' and its proof is omitted for expositional clarity.

above, trivial equilibria always exist. It is also possible that non-stationary equilibria exist over the range of parameter values in question. Nonetheless, it is interesting that within the narrower class of equilibria, existence is restored for even smaller values of ψ. This is because for $\psi \leq \psi_3$, trading only with safe agents can be sustained as an equilibrium. In this event, equilibria exist in which lenders trade only with safer agents and riskier agents are shut out of markets. If ψ is too far below ψ_3, however, existence problems may arise once again, as even safer borrowers default with probability one at $t = 1$.

3.2 Private information.

Suppose lenders cannot identify the output probabilities of individual borrowers. This amounts to assuming that there is only one market for securities at each location, with all securities selling at one price.

The apple producer/lenders' problem is expressed along analogous lines as in the previous sub-section. The difference is that instead of choosing individual quantities of riskier and safer securities, the lender buys undifferentiated securities and chooses $\theta_A(t)$. Let $p(t)$ denote the price of a security bought at time t and $q(t+1)$ the probability with which it repays in the following period. The solution to the lender's problem must satisfy

$$\frac{q(t+1)}{p(t)} \begin{cases} > \\ = \\ < \end{cases} \frac{\alpha}{\beta} \implies \theta_A(t) = \begin{cases} 1 \\ \in [0,1] \\ 0 \end{cases} \tag{11}$$

at each $t = 2, 4, \cdots$.

Banana producer/borrowers follow strategies along the lines developed in the previous sub-section. At time t, $t = 0, 2, \cdots$, borrowers face a common price $p(t)$ of securities and decide on the fraction $\theta^i(t+1)$, $i = \{S, R\}$, of their expected output to exchange. At time $t+1$, they decide on the probability of default $\lambda^i(t+1)$, based on the future path of security prices. The description of a borrower's problem and the results associated with it are analogous to the previous sub-section. $p(t)$, the undifferentiated price of securities,

replaces $p^i(t)$ ($i = \{S, R\}$), the price resulting from common knowledge.

In summary, borrowing is described by:

$$p(t) \begin{cases} > \\ = \\ < \end{cases} \pi^i \beta\alpha(1 - \lambda^i(t+1)) \implies \theta_B^i(t+1) = \begin{cases} 1 \\ \in [0,1] \\ 0; \end{cases} \tag{12}$$

$t = 0, 2, \cdots$.

Considering price sequences that are stationary from time τ onwards at a level p default can be described as follows:

$$p \begin{cases} > \\ = \\ < \end{cases} \hat{p}^i \implies \lambda^i(\tau - 1) = \begin{cases} 0 \\ \in [0,1] \\ 1; \end{cases} \tag{13}$$

where $\hat{p}^i$ is as defined in the previous section, $i = \{S, R\}$. As before, since $\hat{p}^i > \pi^i\beta\alpha$, one implication of equation (12) is that in any borrowing period, if $p(t) \geq \hat{p}^i$, then $\theta_B^i(t+1) = 1$, $i = \{S, R\}$.

The market clearing condition is

$$N_A\theta_A(t) = p(t)N_B\left[\sigma\phi^S(t)\theta_B^S(t+1) + (1-\sigma)\phi^R(t)\theta_B^R(t+1)\right]. \tag{14}$$

The unconditional repayment probability of an individual security is described by

$$q(t+1) = \frac{\pi^S\sigma\phi^S(t)\left[1 - \lambda^S(t+1)\right] + (1-\sigma)\phi^R(t)\left[1 - \lambda^R(t+1)\right]}{\sigma\phi^S(t)\theta_B^S(t+1) + (1-\sigma)\phi^R(t)\theta_B^R(t+1)}. \tag{15}$$

Definition 4 *An equilibrium in the credit economy with private information is described by a sequence of security prices $\{p(2t)\}_{t=0}^{\infty}$, repayment probabilities $\{q(2t+1)\}_{t=0}^{\infty}$, lending decisions $\{\theta_A(2t)\}_{t=0}^{\infty}$, borrowing decisions $\{\theta_B^i(2t+1)\}_{t=0}^{\infty}$ decisions to default $\{\lambda^i(2t+1)\}_{t=0}^{\infty}$ and fractions of eligible borrowers $\{\phi^i(2t)\}_{t=0}^{\infty}$, $i = (S, R)$ that satisfy equations (8), (11)-(15) at each point in time.*

The definitions of *stationary* and *default-free* equilibria follow analogously from the previous sub-section.

Lemma 6 *Given a sequence of stationary prices, $p(t) = p$, starting from some date $t \geq 2$, if at some date $t-1$ along this sequence, $\lambda^S(t-1) \leq 1$, then $\lambda^R(t-1) = 0$; and if $\lambda^R(t-1) \geq 0$, then $\lambda^S(t-1) = 1$.*

Proof: Follows from the inequality, $\hat{p}^S > \hat{p}^R$. If $p \geq \hat{p}^S$, $\lambda^S(t-1) \leq 1$. But then $p > \hat{p}^R$, so that $\lambda^R(t-1) = 0$. And if $p \leq \hat{p}^R$, $\lambda^R(t-1) \geq 0$. But then $p < \hat{p}^S$, $\lambda^S(t-1) = 1$. ■

This establishes that safer borrowers are more likely to default under imperfect information than riskier borrowers. Since their autarkic consumption is higher than that of riskier borrowers, but their income from selling securities is the same, they have less to lose by being excluded from credit markets.

If debt repayments were perfectly enforceable, the only scope for fraudulent behaviour would be to misrepresent endowment probability. Safer borrowers would never gain from such misrepresentation and would, as shown in Jafarey and Rupert (1993), opt out of market transactions altogether for some parameter values rather than face pooling with riskier borrowers. With the possibility of intentional default, and given that the penalty for doing so is no greater than exclusion from future market transactions, there is no incentive for safer borrowers to opt out voluntarily.

Lemma 7 *Given a positive sequence of stationary prices, $p(t) = p > 0$, starting from some $t \geq 2$, no borrower will have an incentive to opt out of issuing debt entirely.*

Proof: Consider the incentive facing a borrower of type i, $i = S, R$. Either: (i) $p \geq \hat{p}^i$, in which case, as Lemma 3 implies, $\theta_B^i = 1$ for all time; or (ii) $p < \hat{p}^i$, in which case a borrower h who has not issued debt before has the incentive, at date t, to sell one unit of security and to follow that with intentional default at $t+1$. The payoff from doing so is bounded below by:

$$p + \frac{\pi^i \beta^2 \alpha}{1 - \beta^2}$$

while the payoff from not issuing a security is

$$\frac{\pi^i \beta^2 \alpha}{1-\beta^2}$$

which is dominated by the payoff from borrowing and defaulting. ■

Hence, limited commitment and private information together create a link between the higher *intrinsic* quality of safer borrowers and their greater tendency to display dishonesty in repayment.

The characterization of equilibria under private information proceeds more or less along the lines developed in the previous section. To summarize, for any $\psi \geq \hat{p}^R(1-\sigma\pi^S)/\eta$, equilibria can be found in which riskier borrowers *never* default. Within this interval, the following thresholds can be defined: $\psi_1^P = \hat{p}^S/\eta$, $\psi_2^P = \hat{p}^S(1-\sigma\pi^S)/\eta$, $\psi_3^P = \hat{p}^S(1-\sigma)/\eta$ and $\psi_4^P = \hat{p}^R(1-\sigma\pi^S)/\eta$.

if $\psi \geq \psi_1^P$, default-free equilibria can be constructed in which both sets of borrowers do not strategically default. Under private information, such equilibria are obviously not the same as under common knowledge. For any value of ψ which results in default-free equilibria in both economies, riskier agents are better off under private information while safer agents are better off under common knowledge. The private information economy results in a *smaller* set of default-free equilibria if and only if the following condition holds:

$$\sigma \leq \frac{\alpha\beta}{\frac{1}{\pi^R}\left[\frac{\alpha}{\beta} - \alpha\beta\right] + \alpha\beta}$$

i.e., the proportion of borrowers who are safer is relatively small. In other words, if the proportion of safer borrowers is large enough, the private information economy results in a larger set of default-free equilibria.

If $\psi \in (\psi_2^P, \psi_1^P]$, equilibria can be stationary from $t=2$ after partial default at $t=1$ by safer agents. Such equilibria in the private information economy are analogous to those that arise in the common knowledge economy for $\psi \leq \psi_1$.

For a lower range of values, $\psi \in [\psi_3^P, \psi_2^P]$, time-$t$ stationary equilibria can be constructed

in which safer borrowers default with probability one at $t = 1$, but the unconditional repayment probability of an undifferentiated security remains positive, backed by the repayment behavior of riskier borrowers. For example, between ψ_2^P and $\psi = \hat{p}^s(1 - \pi^S\sigma(2 - \pi^S)/\eta$, time-4 stationary equilibria can be constructed. In these equilbria, safe borrowers default with probability one at $t = 1$. However, since $1 - \pi^S$ of them do not receive any endowment at $t = 1$ and thus cannot be told apart from unendowed riskier borrowers, they continue to sell securities at $t = 2$ (notwithstanding the fact that had they received an endowment at $t = 1$ they would have defaulted strategically). At $t = 3$, the surviving safer borrowers default with probability less than one, following which no further defaults take place.

For $\psi < \psi_3^P$, security prices can never rise sufficiently to induce safer borrowers to ever repay. This leads to another range of values of ψ in which safer borrowers always default, and get eliminated from borrowing at a rate proportional to π^S of their remaining type every period. Nonetheless, over the interval in question riskier borrowers never default. As shown in the Appendix (Proposition P.1), equilibria can exist with active trade in every period, but these equilibria do not lead to a stationary price sequence, making them non-comparable with the equilibria studied in the common knowledge case.

While the private information market need not result in an equilibrium with active trade over all values of ψ, it is interesting that it can allow for a stationary active equilibrium to exist over parameter values for which the repayment probability of one of the two sub-classes of borrowers is zero, so long as the other sub-class is induced to repay with positive probability. This was not the case in the full information economy. While these results appear in contrast to Smith (1986), where non-existence of equilibrium was associated with private information, it does share a feature of that result: *i.e.* opening an additional market can destroy existence of an equilibrium. In Smith (1986) opening a private information credit market can destroy an autarky equilibrium that exists without any markets, while in the present model opening markets for differentiated securities can destroy an equilibrium of a type which exists under a single undifferentiated market.

4 A Combined Money-Credit Economy.

In this section, the effects of adding money to a credit economy with asymmetric information are considered. Under the model in this paper, a credit market with common knowledge about borrower risk is not compatible with the use of fiat money.[10]

Suppose that at each location, in addition to a market in which banana producers can sell undifferentiated securities to apple producers, there is also a market in which fiat money can be exchanged for goods. Assume that money exists in the amount of M currency units per location at $t = 0$ and is held entirely by banana producers.

Decision making in this economy follows the pattern: each apple producer decides what fraction of his output to sell, splitting this amount between money and securities. Given that a banana producer enters a new location with a positive inventory of money, he can use it to acquire consumption goods in that period. Nonetheless, he still faces a decision: whether to borrow against his *expected* output in the following period or to wait and, conditional upon receiving an output, trade through the money market. If he decides to wait and receives an output in the next period, he sells it for money and moves to another location where he faces the same choice. If he decides to borrow, he decides in the next period, conditional upon output, whether to repay the debt or become a defaulter.

In the presence of an active money market, simply denying access to credit transactions represents a strictly weaker penalty than in the case where only credit transactions were possible. This is because a default could be followed by switching to monetary transactions forever thereafter. Not only would this undermine the feasibility of credit transactions, it would imply that the existence of opportunities for monetary transactions can by itself alter the set of penalties facing loan defaulters. In the spirit of keeping the penalty structure comparable across the two cases, the government is assumed to permanently exclude intentional

[10] It can be shown that an equilibrium cannot exist in which trade takes place in all three assets: fiat money, riskier securities and safer securities. The proof involves showing that the conditions required for such an equilibrium would violate the optimality conditions of at least one set of agents.

defaulters from market transactions.

The problem facing apple producers/lenders is to choose the amount $\theta_A(t)$ of their outputs to trade, allocating a fraction $\xi_A(t)$ of this towards the purchase of securities and a fraction $(1-\xi_A(t))$ towards money. The price of money, denoted by $p^M(t)$, is measured in units of time-t good exchanged for one unit of currency. The price of securities, denoted by $p(t)$, measures units of time-t good exchanged for the promise of one unit of time-$t+1$ good. The recursive problem facing apple producers in each even-numbered period can thus be written:

$$\begin{aligned} V_A(t) = \max_{\theta_A(t),\xi_A(t)} \Bigg\{ & \alpha(1-\theta_A(t)) \\ & + \beta\left[\xi_A(t)\frac{q(t+1)}{p(t)} + (1-\xi_A(t))\frac{p^M(t+1)}{p^M(t)}\right]\theta_A(t) \\ & + \beta^2 V_A(t+2)\Bigg\} \\ s.t.: \quad & \theta_A(t) \leq 1; \end{aligned}$$

for $t = 0, 2, \cdots$. This formulation of the apple producers' problem imposes the implicit restriction that money is never hoarded, *i.e.* all money acquired in one period is used to purchase goods in the next. Given discounting and risk neutrality, the only motive for hoarding money in this model would be if its value was expected to appreciate over time. Our interest is in stationary equilibria in which prices stay constant over time; in such equilibria there is no incentive to hoard money.[11]

The solution to the apple producers' problem satisfies

$$\frac{p^M(t+1)}{p^M(t)} = \frac{q(t+1)}{p(t)} \Longrightarrow \xi_A(t) \in [0,1] \tag{16}$$

$$\frac{p^M(t+1)}{p^M(t)} \begin{cases} < \\ = \\ > \end{cases} \frac{\alpha}{\beta} \Longrightarrow \theta_A(t) = \begin{cases} 0 \\ \in [0,1] \\ 1 \end{cases} \tag{17}$$

[11] In a pure monetary version of this economy, it has been shown that equilibria in which the value of money *falls over time are possible but equilibria in which it rises are not possible.*

Equation (16) requires both assets to yield the same expected return to an apple producer/lender. It a necessary condition for the existence of an equilibrium in which both money and securities are traded. Equation (17) requires the return on trade of any sort to be at least as great as that from autarkic consumption.

Any banana producer who enters the apple season with money uses it to buy apples. He can nonetheless acquire more apples by issuing a security. For notational convenience, we assume that if a banana producer feels indifferent between issuing a security at time t or waiting for an output and trading for money at time $t+1$, he opts for one or the other and does not do a bit of both. Hence, banana producers/borrowers face the following decision in each even-numbered period.

$$V_B^i(t) = \max\left[V_B^{M,i}(t), V_B^{C,i}(t)\right] \tag{18}$$

where:

$$\begin{aligned} V_B^{M,i}(t) &= \underset{\theta_B^{M,i}(t+1)}{\operatorname{argmax}} \left\{\pi^i\beta\left[\alpha(1-\theta_B^{M,i}(t+1)) + \beta\frac{p^M(t+2)}{p^M(t+1)}\theta_B^{M,i}(t+1)\right] + \beta^2 V_B^i(t+2)\right\} \\ s.t.: &\quad 0 \le \theta_B^{M,i}(t+1) \le 1. \end{aligned}$$

$$\begin{aligned} V_B^{C,i}(t) = &\underset{\theta_B^{C,i}(t+1),\lambda^i(t+1)}{\operatorname{argmax}} \Bigg\{p(t)\theta_B^{C,i}(t+1) \\ &+ \beta\pi^i\left[\lambda^i(t+1)\tilde{V}_B^i + (1-\lambda^i(t+1))\left(\alpha(1-\theta_B^{C,i}(t+1)) + \beta V_B^i(t+2)\right)\right] \\ &+ (1-\pi^i)\beta^2 V_B^i(t+2)\Bigg\} \\ s.t.: &\quad 0 \le \theta_B^{C,i}(t+1) \le 1; \end{aligned}$$

for $t = 0, 2, \cdots$. $\theta^{M,i}$ and $\theta^{C,i}$ denote the proportion of endowment traded given the choice of going through the money or credit market, respectively. $\tilde{V}_B^i$ is the payoff from defaulting intentionally, as described in Section 3.1.

Definition 5 *An equilibrium in the combined money and credit market is described by sequences of security prices $p(2t)$, currency prices $p^M(t)$, repayment probabilities $q(2t+1)$,*

decisions to trade, $\theta_A(2t)$, $\xi_A(2t)$, $\theta_B^{M,i}(2t+1)$, $\theta_B^{C,i}(2t+1)$, *decisions to default* $\lambda^i(2t+1)$ *and fractions of eligible borrowers* $\phi^i(2t)$, $i = (S, R)$, $t \geq 0$, *that satisfy equations (16) and (17), solve (18) and lead to market-clearing in both the money market and the security market at each point of time.*

The definitions of time-t stationary and default-free equilibria follow analogously from above. To avoid cumbersome detail, we have not yet described the solution to banana producer/borrowers' problems or the market clearing conditions. These tasks become easier if we restrict attention to the case of equilibria which become stationary at some time. Let p, $p^M(f)$ and $p^M(s)$ denote the stationary prices of securities and money in fall and summer, respectively. Also, note that in a stationary equilibrium the rate of intentional default has to be zero for both types of borrowers. If, in addition, a positive fraction of borrowers of type i remain eligible to participate in trade, it must be the case that $V_B^{C,i} \geq \tilde{V}_B^i$. Also, Lemma 3 implies that, in this case, $\theta_B^{C,i} = 1$, given that a banana producer chooses to trade through the credit market. In this environment, we can write the following stationary value functions for banana producers:

$$V_B^{M,i} = \pi^i \beta \left[\alpha(1 - \theta_B^{M,i}) + \beta \frac{p^M(f)}{p^M(s)} \theta_B^{M,i} \right] + \beta^2 V_B^i$$
$$V_B^{C,i} = p + \beta^2 V_B^i$$

where $V_B^i = \max\,[V_B^{M,i}, V_B^{C,i}]$. It is easy to see that, given a banana producer who has chosen to trade through the money market:

$$\frac{p^M(f)}{p^M(s)} \begin{cases} > \\ = \\ < \end{cases} \frac{\alpha}{\beta} \implies \theta_B^{M,i} = \begin{cases} 1 \\ \in [0,1] \\ 0 \end{cases} \tag{19}$$

and, therefore, in comparing the payoffs for banana producers from each type of transaction

$$V_B^{C,i} \begin{cases} > \\ = \\ < \end{cases} V_B^{M,i} \iff p \begin{cases} > \\ = \\ < \end{cases} \pi^i \beta^2 \frac{p^M(f)}{p^M(s)} \theta_B^{M,i} \tag{20}$$

for $i = \{S, R\}$.

The following describes credit market clearing, which takes place every two periods:

$$N_A \xi_A \theta_A = p \cdot N_B \left[\sigma \chi^S \pi^S + (1-\sigma)\chi^R \phi^R\right] \tag{21}$$

where χ^S, χ^R denote fractions of banana producers who trade through credit. The money market conditions are:

$$p^M(f)M = N_A(1-\xi_A)\theta_A \tag{22}$$

$$p^M(s)M = N_B\left[\pi^S \sigma(1-\chi^S)\phi^S \theta_B^{M,S} + \pi^R(1-\sigma)(1-\chi^R)\phi^R \theta_B^{M,R}\right] \tag{23}$$

Hence, in a stationary environment:

$$p = \frac{\psi\eta\xi_A\theta_A}{\sigma\chi^S\phi^S + (1-\sigma)\chi^R}\phi^R \tag{24}$$

$$\frac{p^M(f)}{p^M(s)} = \frac{\psi\eta(1-\xi_A)\theta_A}{\pi^S\sigma(1-\chi^S)\phi^S\theta_B^{M,S} + \pi^R(1-\sigma)(1-\chi^R)\phi^R\theta_B^{M,R}} \tag{25}$$

Lemma 8 *If both money and credit are used in a stationary environment, riskier borrowers will strictly prefer to trade through credit.*

Proof: For lenders to use both markets in a stationary environment, equation (16) implies that:

$$\frac{p^M(s)}{p^M(f)} = \frac{q}{p}$$

where $q \geq \pi^R$. Hence $p = q \cdot \left(p^M(f)/p^M(s)\right)$. For riskier borrowers to be willing to use the money market, it is *necessary* that $p \leq \pi^R\beta^2 \cdot \left(p^M(f)/p^M(s)\right)$. But since $q \geq \pi^R$ and $\beta < 1$, this is not possible. ■

Hence, if money and credit are to coexist, at least some safer agents must be willing to use the money market. The following assumption allows for separating equilibria to arise in which safer agents strictly prefer trading through money instead of credit:

Assumption 4 $\beta^2\pi^S > \pi^R$.

Assumption 4 suggests that safer borrowers are motivated more by the difference between their output probabilities and those of riskier agents than they are by the impatience to consume now rather than later. Note that Assumption 4 does not rule out the possibility of equilibria in which safer agents use both money and credit.

Under Assumption 4, provided that separation does occur, the private information credit economy with money generates equilibria that parallel those of the common knowledge credit economy. This is not surprising, since separation implies that those banana producers who do borrow in equilibrium identify themselves as riskier ones. Referring to the threshold values defined in Section 3.1, it can be shown that for each value of $\psi \in [\psi_1, \beta/\alpha]$, a default-free equilibrium exists in which safer banana producers trade their entire output for money while riskier ones trade their entire expected output for credit and never default (see Lemma M.1, Appendix). Similarly, for $\psi \in [\psi_2, \psi_1)$, there is an equilibrium in which separation continues to occur as described above, but a fraction of riskier borrowers default at $t = 1$ while the remaining fraction proceed to repay their debts consistently thereafter (see Lemma M.2, Appendix).

Recall that once $\psi \leq \psi_2$, there exist intervals of values for which the common knowledge credit economy results in no trade taking place. With the addition of money to a credit economy, such non-existence becomes unlikely since for parameter values under which the credit economy does not function well, trades can still go through money. The next proposition covers these cases.

Proposition 2 *For every $\psi \leq \psi_2$ there is a stationary equilibrium from $t = 0$ in which trades take place through money.*

We now turn to the possibility of multiple equilibria. Trivially, for any parameter value, there may be equilibria in which only monetary trades take place or, conversely, only credit trades take place. More significantly, there may be equilibria in which both money and credit are used, but there is no separation of riskier and safer borrowers in the credit market. For

example, suppose the following restrictions hold:

Assumption 5 $\alpha < \beta^2$.

Assumption 6 $\pi^S > \alpha^2(1-\beta^2)/[\beta^2(\beta^2-\alpha^2)]$.

Assumption 7 $\sigma > (\beta^2\pi^S - \pi^R)/[\pi^S - \pi^R]$

Assumption 7 ensures that $\tilde{p}^S < \beta^3/\alpha$. This is only to ensure that the equilibria referred to in the following proposition are consistent with $\theta_A = 1$. The other two assumptions ensure that the values of π^S and χ^S needed for the following proposition to hold are both feasible, *i.e.* below unity.

Proposition 3 *For every value of $\psi \in [\tilde{p}^S/\beta^2, \beta/\alpha]$, there exists a default-free stationary equilibrium from $t = 0$ in which banana producers trade their full output with riskier ones trading only through credit and safer ones trading through both money and credit.*

Assumptions 3-7, have been made in the spirit of identifying regions of parameter space over which the stated results hold. It is easy to show that this space is non-empty: *e.g.*, $\alpha = 0.2$, $\beta = 0.9$, $\pi^S = 0.8$, $\pi^R = 0.1$ and $\sigma = 0.8$. In particular, Assumptions 5-7 can hold along with Assumption 4 so that multiple stationary equilbria can exist once money is introduced along with credit.

The reason for multiple equilibria appears to be a strategic complementarity among safer borrowers. If safer borrowers enter the credit market, there are two effects. All else equal, the price of securities falls due to a larger number of borrowers. This tends to discourage entry by safer borrowers. At the same time, the creditworthiness of the pool of borrowers improves. This makes lenders more willing to participate in the security market and raises the price of securities while lowering the terms of trade for borrowers in the money market. The second effect encourages entry by safer borrowers. The possibility of multiple equilibria suggests that revelation of borrower type is a somewhat tenuous outcome of introducing money into a pure credit economy.

To summarise, adding money leads to equilibria involving trade for a larger set of parameter values than happened in the credit economy alone. This suggests that adding money to a private information credit economy serves two purposes: (i) it can expand the set of parameter values over which equilibria involving trade take place; (ii) it can potentially reveal borrower risk in credit markets, provided separation takes place.[12]

5 Conclusion

A model in which private liabilities can be used to facilitate intertemporal trade, but suffer from limited commitment and adverse selection, was studied. It was shown that when combined with adverse selection the limited commitment problem affects the behavior of safer borrowers more severely than that of riskier ones. By contrast, in the absence of adverse selection the limited commitment problem affects riskier borrowers to a greater extent. These results suggest that strategic default does not correspond uniquely to inherent ability to repay; therefore, default behavior can go either way in weeding out different classes of borrowers.

It was also shown that a credit market may perform worse under common knowledge than under private information. In particular, a common knowledge credit market may fail to deliver a non-trivial equilibrium of a type that could exist in an analogous situation under private information. Hence, given imperfect enforcement, alleviating informational frictions need not improve the functioning of credit markets.

Adding fiat currency was found to ease frictions associated with a credit market based on private information. In particular, the combination of adverse selection and limited commitment which especially affected higher quality borrowers was potentially alleviated. Separating equilibria arose in which the latter opted out of credit transactions entirely rather

[12] To our knowledge, this role was originally identified by Smith (1986)). Smith also found that money can expand the set of parameter values over which trade takes place, but this result pertained to the case in which credit transactions take place under asymmetric information, while the present result also applies to the common knowledge case.

than face pooling with riskier borrowers. Such equilibria were analogous to the equilibria of a credit economy based on common knowledge, except that over parameter values for which the credit market would become inactive, equilibria with trade could exist in the monetary economy. The informational role of money is tenuous, however, as pooling equilibria in which safer borrowers continue to use credit can coexist with revealing ones.

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6 Appendix

Proposition F.1: For each value of $\psi \in [\psi_1, \beta/\alpha]$, there exists a default-free equilibrium in which all agents trade their full endowments.

Proof: Given that equation (3) is met in equilibrium, we can express the price of each security in terms of a fictitious numeraire:

$$p(t) = \frac{p^i(t+1)}{q^i(t)} \quad i = S, R;\ t = 0, 2, \ldots \tag{26}$$

With this modification, we can express the lenders' optimality condition in terms of the numeraire

$$\frac{1}{p(t)} \ > \ (=) \ \{<\} \ \frac{\alpha}{\beta} \ \implies \ \theta_A(t) = 1 \ (\in \ [0,1]) \ \{= 0\} fs.$$

where $\theta_A(t) \equiv \theta_A^S(t) + \theta_A^R(t)$. We can use the above to aggegate the market-clearing conditions

$$N_A \theta_A(t) = N_B p(t) \left[\sigma \phi^S(t) \pi^S \lambda^S(t+1) \theta_B^S(t+1) + (1-\sigma) \phi^R(t) \pi^R \lambda^R(t+1) \theta_B^R(t+1)\right]$$

Note that $\psi_1 = \tilde{p}^R$ and that $\tilde{p}^R - \alpha/\beta = [(\alpha/\beta) - \alpha\beta][(1/\pi^R) - 1] \geq 0$ implying that $\psi \geq \alpha/\beta$ over the entire interval. Start with the conjecture that over the interval, there exist stationary, default-free equilibria involving full participation by both types of producers. Following this conjecture, set $\phi^i = 1$, $\lambda^i = 0$, $\theta_A = 1$ and recall from Lemma 3 that, in any default-free equilibrium $\theta_B^i = 1$, $i = (S, R)$. From the market-clearing condition,

$$N_A = N_B p \left[\pi^S \sigma + \pi^R (1-\sigma)\right] \implies p = \psi$$

Since $\psi \in [\tilde{p}^R, \beta/\alpha]$, this implies $p \leq \beta/\alpha$ over the interval, therefore from the lender's incentives, $\theta_A = 1$ as conjectured. It also implies that $p \geq \tilde{p}^R > \tilde{p}^S$, so that the numeraire price remains high enough to deter default. The conjecture $\lambda^i = 0$ and $\phi^i = 1$, $i = (S, R)$ is verified and from Lemma 3, so is the conjecture that $\theta_B^i = 1$, $i = S, R$. ■

Proposition F.2: Over the interval, $(\psi_2, \psi_1]$, there is a unique value of ϕ^R such that if $\phi^R(0) = \phi^R$, a default-free stationary equilibrium exists with $p = \tilde{p}^R$. In addition, there is

an open subset of values of ψ below ψ_1 such that, given $\phi^R(0) = 1$, there is a unique rate of default by risky borrowers at $t = 1$, which leads to a time-2 stationary equilibrium.

Proof: Conjecture that a stationary default-free equilibrium exists with $\theta_B^R = \theta_B^S = \theta_A = 1$ and $p = \tilde{p}^R$. Arbitrage between the two types of securities requires that $p = p^S/\pi^S = p^R/\pi^R$.

$$\begin{aligned} N_A &= N_B \left[p^S\sigma + p^R(1-\sigma)\phi^R\right] \\ &= N_B p \left[\pi^S\sigma + \pi^R(1-\sigma)\phi^R\right]. \end{aligned}$$

Plugging in the conjectured value of p, the above may be solved for a unique value ϕ^R:

$$\phi^R = \frac{1}{\pi^R(1-\sigma)}\left[\frac{\eta\psi}{p} - \pi^S\sigma\right]$$

which reduces to $\phi^R = 1$ when $\psi = \tilde{p}^R$. Hence, given $\phi^R(0) = \phi^R$ a stationary price path exists with $p = \tilde{p}^R$. Given this price, the conjectured values of $\theta_A = \theta_B^i = 1$, and $\lambda^i = 0$, $i = S, R$ are all verified.

Starting from $\phi^R(0) = 1$, the question arises whether the critical value ϕ^R can be reached after default by some risky agents at $t = 1$ or thereafter. Indeed, over the interval in question, there is a unique rate of default at $t = 1$:

$$\lambda^R(1) = \frac{1-\phi^R}{\pi^R} = \frac{\eta}{(\pi^R)^2(1-\sigma)}\left[\frac{\eta\psi}{\tilde{p}^R - \pi^S\sigma}\right]$$

such that $\phi^R(2) = \phi^R$ and a default-free stationary equilibrium can exist from $t = 2$ onwards with $p(2) = p(4) = \cdots = \tilde{p}^R$. For the above value of $\lambda^R(1)$ to be an equilibrium from $t = 0$, it is necessary that (i) trade takes place in both types of securities at $t = 0$, (ii) safe agents do not default at $t = 1$, (iii) risky agents default with the above probability.

Given the equilibrium from $t = 2$ and noting that default behavior depends on future, not past, prices, $p(2t) = \tilde{p}^R$, $t \geq 1$ ensures that, given Lemmas 1 and 5, safe agents do not default at $t = 1$ while risky ones default with any probability.

We need to verify that agents remain willing to trade securities at $t = 0$. The numeraire price at $t = 0$ (after accounting for the arbitrage condition across securities) satisfies:

$$N_A\theta_A(0) = N_B p(0)[\pi^S\sigma\theta_B^S(1) + \pi^R(1-\lambda^R(1))(1-\sigma)\theta_B^R(1)]$$

Hence,

$$p(0) = \frac{\eta\psi\theta_A(0)}{\pi^S\sigma\theta_B^S(0) + \pi^R(1-\lambda^R(1))(1-\sigma)\theta_B^R(0)}$$

At $\psi = \tilde{p}^R$, $p(0) = \tilde{p}^R$ arises if we assume $\theta_A(0) = \theta_B^i(1) = 1$, $\lambda^R(1) = 0$. Note that $\pi^i\tilde{p}^R > \pi^i\alpha\beta$, $i = S, R$ and that by Assumption (3), $\tilde{p}^R < \beta/\alpha$. Hence, the assumed choices of θ are optimal. By continuity of $p(0)$ in ψ and $\lambda^R(1)$, and of $\lambda^R(1)$ in ψ, there is an open interval of values of ψ below $\tilde{p}^R$, such that $\beta\alpha \leq p(0) \leq \beta/\alpha$ and the same values of θ continue being optimal. ■

Lemma F.1: Suppose there exists a critical value ϕ^R which supports a stationary equilibrium with $p^R = \hat{p}^R$. Given that $\phi^S(t) = 1$ and $\lambda^S(t+1) = 0 \ \forall \ t = 0, 2, 4, \cdots$, $\phi^R(t) > \phi^R \ \Rightarrow \ p^R(t) < \hat{p}^R \ \ t = 2, 4, \ldots$.

Proof: The market clearing condition (defined in terms of the numeraire security; see the proof of Proposition F.1) modifies to:

$$N_A\theta_A(t) = N_B p(t)\left[\pi^S\sigma + \pi^R(1-\lambda^R(t+1))(1-\sigma)\phi^R(t)\right]$$

which, upon taking account of the indifference of lenders between the two types of securities can be expressed in terms of $p^R(t)$:

$$N_A\theta_A(t) = N_B p^R(t)\left[\frac{\pi^S}{\pi^R(1-\lambda^R(t+1))}\sigma + (1-\sigma)\phi^R(t)\right]$$

which may be solved for $p^R(t)$:

$$p^R(t) = \frac{\psi\eta\theta_A(t)}{Z}$$

where

$$Z = \frac{\pi^S}{\pi^R(1-\lambda^R(t+1))}\sigma + (1-\sigma)\phi^R(t)$$

By construction, if $\phi^R(t) = \phi^R$, then $\lambda^R(t+1) = 0$, $p^R(t) = \hat{p}^R$ and $\theta_A(t) = 1$. For $\phi^R(t) > \phi^R$, $\lambda^R(t+1) \geq 0$ and $\theta_A(t) \leq 1$. Therefore $p^R(t) < \hat{p}^R$. ■

Proof of Proposition 1:

Proof: For ψ in the interval in question, a critical value ϕ^R which supports a stationary equilibrium with $p^R = \hat{p}^R$ still exists, but is so small that $\phi^R(2) > \phi^R$ even if $\lambda^R(1) = 1$. This can be checked from the expression for ϕ^R as given in the proof of Proposition F.2. The question arises whether the stationary value of ϕ^R needed to sustain $p^R = \hat{p}^R$ can be reached over more than one round of default.

Suppose that an equilibrium exists in which ϕ^R is reached at $t = 4$ or later. Note that $\lambda^R(1) = 1 \;\Rightarrow\; p^R(0) = 0$. Receiving a zero price for securities at $t = 0$ means that borrowers cannot be in default at $t = 1$, even if they had chosen to sell at this price. Hence, $\lambda^R(1) < 1$ is a necessary condition for reaching $\phi^R < 1$. This implies that $\{p^R(2t)\}_{t=1}^{\infty}$ is high enough to prevent *certain* default at $t = 1$ by riskier borrowers. Lemma F.1 above shows that $\phi^R(2) > \phi^R \;\Rightarrow\; p^R(2) < \hat{p}^R$.

Suppose, then, that $\lambda^R(1) < 1$ so that $p^R(2) < \hat{p}^R$. Trivially, two possibilities exist.

One is that riskier borrowers *do not* strictly prefer to pay back their debts at $t = 3$. In this case, given Lemmas 1 and 2 and since $p^R(2) < \hat{p}^R$, they would have strictly preferred to default at $t = 1$, contradicting the assumption that $\lambda^R(1) < 1$.

Alternatively, suppose that riskier borrowers *do* strictly prefer to pay back at $t = 3$. This could be consistent with $\lambda^R(1) < 1$, but it would also imply that $\lambda^R(3) = 0$. But then $\phi^R(4) = \phi^R(2)$. By iterating the argument, $\phi^R(2t)$ remains constant for all $t \geq 1$ at the level $\phi^R(2)$. But this cannot be a stationary equilibrium since $\phi^R(2) > \phi^R$, and ϕ^R is the unique value that can sustain a stationary price, $\hat{p}^R$, for riskier securities. Hence, the assumed equilibrium fails to exist.

Can there be an equilibrium in which lenders buy securities only from safer borrowers? Suppose there is. Market-clearing implies:

$$N(1)\theta_A(t) = p^S(t)N(2)\sigma\theta_B^S(t)$$

Therefore,

$$p^S(t) = \frac{\psi\eta}{\sigma}\frac{\theta_A(t)}{\theta_B^S(t)}$$

Over the relevant range of values of ψ, if an equilibrium exists in which lenders only lend to safer borrowers, then $\theta_A(t) = \theta_B^S(t) = 1$ $t = 0, 2, \ldots$. This implies that $p^S(t) = p^S = \psi/\sigma$. But then, an isolated riskier borrower who receives a sequence of loans at the stationary price $(p^R =)$ $(\pi^R/\pi^S)p^S$ will have an incentive to repay at every stage. This is because

$$\frac{pi^R}{\pi^S}p^S > \hat{p}^R \implies \frac{\psi\eta}{\pi^S\sigma} > \tilde{p}^R$$

which is true by definition throughout the interval. such a transaction would be strictly preferred by lenders. Hence, there can be no equilibrium in which lenders transact only with safer agents.

Ruling out a no-trade equilibrium follows the lines along which an equilibrium with only safer securities was ruled out. Lenders are willing to pay up to $\eta\beta/\alpha$ for securities so long as intentional default does not take place. At this price and over the interval in question , it is feasible to have trades involving either type of borrower which result in a zero probability of intentional default. ■

Proposition P.1: Suppose that $\pi^R > \alpha^2(1 - \beta^2(1 - \pi^S))/\beta^2$. This is a sufficient condition for active equilibria to exist over the interval, $\psi \in [\psi_4^P, \psi_3^P]$, in which these equilibria no risky borrower ever defaults, while safe ones always do.

Proof: Note that for each value of ψ in the interval, the price must remain within the interval $[\hat{p}^R, \hat{p}^S]$ for all time, in order for the stated equilibrium to exist.

We start by conjecturing that $\theta_A(2s) = 1$ for all $s = 0, 1, \cdots$. If so, given the other conjectures made, the market clearing condtion would be:

$$N_A = N_B\left[(1-\sigma) + \sigma\phi^S(2s)\right]p(2s)$$

with $\phi^S(2s) = (1-\pi^S)^s$ by iterative application of equation (8). Hence,

$$p(2s) = \frac{\psi\eta}{(1-\sigma)+\sigma(1-\pi^S)^s}$$

and $p(2s)$ rises with s.

The relevant initial period to study default behavior is $t = 2$ (or $s = 1$). For any value of ψ, in the conjectured equilibrium,

$$p(2) = \frac{\psi\eta}{(1-\sigma)+\sigma(1-\pi^S)} = \frac{\psi\eta}{1-\sigma\pi^S}$$

Defining $\underline{p}(2)$ as the infimum of $p(2)$ over the interval of values of ψ in question, it is easy to see that this arises at ψ_4^P. Imposing the definition of ψ_4^P, this shows that $\underline{p}(2) = \hat{p}^R$. This establishes, that $p(t) \geq \hat{p}^R$ for each value of ψ over the entire interval.

Now note that as $s \longrightarrow \infty$, $p(2s) \longrightarrow (\psi\eta)/(1-\sigma)$ from below. Let p_∞ denote the limiting value of $p(2s)$ as s approaches infinity, and let $\bar{p}_\infty$ denote the supremum of p_∞ over the interval in question. Since $p(2s)$ increases in ψ for any s, $\bar{p}_\infty$ is attained at ψ_3^P. Plugging in the value of ψ_3^P, it is seen that $\bar{p}_\infty = \hat{p}^S$.

This verifies the conjectures that safer borrowers always, and riskier borrowers never, default strategically over the entire time path associated with each value of ψ in the interval.

We need to verify the conjecture that $\theta_A(2s) = 1$. This requires that

$$\frac{q(2s+1)}{p2s} \geq \frac{\alpha}{\beta}$$

In the conjectured equilibrium,

$$q(2s+1) = \frac{(1-\sigma)\pi^R}{(1-\sigma)+\sigma(1-\pi^S)^s}$$

Given the expression for $p(2s)$ from above, this gives:

$$\frac{q(2s+1)}{p2s} = \frac{(1-\sigma)\pi^R}{\psi\eta}$$

which is independent of the time path and increases as ψ decreases. A sufficient condition for the conjecture of θ_A to be verified for the entire interval is that it holds at ψ_3^P. Plugging

in the value, this requires

$$\frac{\pi^R}{\hat{p}^S} > \frac{\alpha}{\beta}$$

Taking into account the definition of $\hat{p}^S$, it is straightforward to see that the assumption on π^R is necessary and sufficient for the above inequality.

Note that the assumption on π^R in the proposition is stronger than Assumption 3 in the text. The conjectured equilibrium could still be proven without this, but the simplifying conjecture of $\theta_A(2s) = 1$ over all relevant cases would have to be discarded. ■

Lemma M.1: For every value of $\psi \in [\tilde{p}^R, \beta/\alpha]$, there exists a default-free stationary equilibrium from $t = 0$ in the combined money-credit economy, in which borrowers trade all their outputs, with riskier borrowers trading through credit and safer ones trading through money.

Proof: Let us start by accepting the conjectures that $\chi^S = 0$, $\chi^R = 1$, $\lambda^R = 0$ and $\theta_B^{M,S} = \theta_B^{C,R} = \theta_A = 1$. From equations (24), (25) after accounting for equation (16, all in the main text, an expression for ξ_A can be derived

$$\xi_A = \frac{\pi^R(1-\sigma}{\eta}$$

Plugging this back into equations (24), (25) gives that $p = \pi^R\psi$, $p^M(f)/p^M(s) = \psi$.

Given these prices, a safer banana producer who goes through money faces the *ex ante* payoff:

$$\beta^2\pi^S\psi$$

while going through securities entails the payoff (since lenders must be made indifferent):

$$\pi^R\psi$$

Under assumption $(A.4)$, therefore, safer agents strictly prefer money. This confirms that $\chi^S = 0$. Also, *given* an output, the return to safer agents from trading is:

$$\beta\psi$$

Since $\beta\psi \geq \beta\tilde{p}^R \geq \alpha$, this is sufficient to ensure that $\theta_B^S = 1$.

In the conjectured equilibrium, apple producers/lenders face the return

$$\frac{1}{\psi}$$

no matter which market they use. Given that $\psi \leq \beta/\alpha$, the return $1/\psi \geq \alpha/\beta$, which verifies that $\theta_A = 1$.

Finally, given Lemma 8, we know that riskier borrowers only trade through securities. This gives them a payoff of

$$\pi^R\psi$$

Since $\psi \geq \tilde{p}^R$ $(= \pi^R\hat{p}^R)$,

$$\pi^R\psi \geq \hat{p}^R$$

This ensures that $V_B^{C,R} > \tilde{V}_B^R$. Hence, riskier agents have no incentive to default at any time. This also verifies the conjectured value $\theta_B^{C,R} = 1$. ■

Lemma M.2: Over the interval, $((1-((1-\sigma)(\pi^R)^2/\eta))\tilde{p}^R, \tilde{p}^R)$, , there is an open subset of values of ψ below ψ_1, for each of which there exists an equilibrium in which riskier borrowers trade through credit and safer ones trade through money, riskier borrowers default with positive probability at $t = 1$ and repay honestly from $t = 3$ onwards.

Proof: We shall begin by assuming that agents behave as conjectured in the proposition and, moreover, that all agents always trade their full endowments. With these conjectures, solve for the values of ξ_A and ϕ^R that support the stated equilibrium from $t = 2$.

Using the market-clearing conditions for money and securities, as well as the indifference condition for apple producers,

$$\frac{\psi\eta\xi_A}{(1-\sigma)\phi^R\pi^R} = \frac{\psi\eta(1-\xi 1)}{\sigma\pi^S},$$

which may be solved for a unique value of ξ_A (given ϕ^R),

$$\xi_A = \frac{(1-\sigma)\phi^R\pi^R}{\sigma\pi^S + (1-\sigma)\phi^R\pi^R}$$

Given Lemma 1, the price of securities in the conjectured equilibrium from $t = 2$ on must equal $\hat{p}^R$. This implies that

$$\hat{p}^R = \frac{\psi\eta\xi_A}{(1-\sigma)\phi^R\pi^R}$$

Plugging in the value of ξ_A from above yields a unique expression for ϕ^R (given ϕ).

$$\phi^R = \frac{1}{\pi^R(1-\sigma)}\left[\frac{\eta\psi}{\tilde{p}^R} - \pi^S\sigma\right]$$

For $\phi^R(2)$ to equal ϕ^R, given that $\phi^R = 1$, the default rate at $t = 1$ must be

$$\lambda^R(1) = \frac{\eta}{(\pi^R)^2(1-\sigma)}\left[1 - \frac{\psi}{\tilde{p}^R}\right].$$

Given these parameters, the price of securities over the entire interval equals $\hat{p}^R$ from $t = 2$ on. The return to apple producers, whether they trade through money or securities, equals $1/\tilde{p}^R > \alpha/\beta$. This rationalises both their indifference towards the two markets and their choice of $\theta_A = 1$.

The expected return to safer banana producers from waiting for a monetary trade equals $\beta^2\pi^S\tilde{p}^R$. If an isolated safer agent were to try and trade through securities, he would only receive $\hat{p}^R = \pi^R\tilde{p}^R$. Since, by assumption, $\beta^2\pi^S > \pi^R$, a safer agent strictly prefers monetary trade. Also, since $\tilde{p}^R > \alpha/\beta$, when a safer agent does receive an output, he trades it all for money ($\theta_B^{M,S} = 1$).

As for riskier banana producers, since $p = \hat{p}^R$, they remain indifferent between defaulting or not as of $t = 1$. Their preference for credit trades and their choice of $\theta_B^R = 1$ follow from previous results, *i.e.* Lemmas 8 and 3.

We now have to verify behavior at $t = 0$. Under the conjecture that riskier agents alone use credit at $t = 0$ and promise their full outputs ($\theta_B^{C,R}(1) = 1$), the price of securities at

$t = 0$ is:

$$p(0) = \frac{\eta\psi\xi_A(0)\theta_A(0)}{1-\sigma}.$$

Under the conjecture that safer agents all wait and trade their full outputs at $t = 1$, the return on money between $t = 0$ and $t = 1$ is

$$\frac{p^M(1)}{p^M(0)} = \frac{\sigma\pi^S}{\eta\psi[1-\xi_A(0)]\theta_A(0)}$$

Apple producers can be made indifferent between money and securities so long as

$$\frac{p^M(1)}{p^M(0)} = \frac{q(1)}{p(0)}$$

which can be used to solve for

$$\xi_A(0) = \frac{(1-\sigma)q(1)}{\sigma\pi^S + (1-\sigma)q(1)}$$

where:

$$q(1) = \pi^R[1-\lambda^R(1)]$$

and $\lambda^R(1)$ has already been solved for.

Once indifference between money and securities is satisfied for apple producers, their aggregate trade, $\theta_A(0)$ will adjust to ensure equation (17) is satisfied. Hence, both (16) and (17) can be satisfied for apple producers.

As for safer banana producers, if they wait for a monetary trade at $t = 1$, their expected return (at $t = 0$) is:

$$\beta^2\pi^S\frac{\psi\eta(1-\xi_A)}{\sigma\pi^S} = \beta^2\pi^S\tilde{p}^R$$

where the equality arises because, from $t = 2$ onwards, the economy will be in a stationary equilibrium as described in the proposition. If they choose to issue securities at $t = 0$, they receive $p(0)$. This entails the inequality (since $\theta_A(0) \leq 1$)

$$p(0) \leq \frac{\eta\psi\xi_A(0)}{1-\sigma}.$$

It can be shown that since $\psi \leq \tilde{p}^R$, $q(1) \leq \pi^R$ and $\xi_A(0) \leq \pi^R(1-\sigma)/\eta$. Hence $p(0) \leq \pi^R\psi \leq \pi^R\tilde{p}^R < \beta^2\pi^S\tilde{p}^R$. Hence, safer agents prefer to wait at $t = 0$ and go through the money market at $t = 1$. Also, it is easy to show that $\beta\pi^S\tilde{p}^R > \alpha$, implying that $\theta_B^{M,S}(1) = 1$ as conjectured.

Turning to riskier banana producers, we have to verify they prefer to issue securities at $t = 0$ than to wait and go through money at $t = 1$. Note that under the conjectured equilibrium $V_B^{C,R}(0)$ is equal to

$$V_B^{C,R}(0) = p(0) + \frac{\beta^2\hat{p}^R}{1-\beta^2}.$$

If they decide to go through the money market instead, given its stationarity from $t = 2$ on, they face

$$V_B^{M,R}(0) = \frac{\beta^2\pi^R}{1-\beta^2}\frac{p^M(f)}{p^M(0)}$$

But, the lenders' incentive condition requires that $\pi^R p^M(f)/p^M(s) = \hat{p}^R$. Hence,

$$V_B^{M,R}(0) = \frac{\beta^2}{1-\beta^2}\hat{p}^R$$

which is dominated by $V_B^{C,R}(0)$ for any $p(0) > 0$.

Finally, we need to verify that $\theta_B^{C,R}(1) = 1$ (as conjectured in obtaining the expression for $p(0)$). Note that, after plugging in the conjectured values of $\xi_A(0)$ and $q(1)$, we get

$$p(0) = \eta\psi\theta_A(0)\frac{\pi^R[1-\lambda^R(1)]}{\sigma\pi^S + (1-\sigma)\pi^R[1-\lambda^R(1)]}$$

Note that, from Lemma M.1, at $\psi = \tilde{p}^R$, $\lambda^R(1) = 0$, and $p(0) = \pi^R\tilde{p}^R$ is consistent with $\theta_A(0) = 1$ and $.\theta_B^{C,R}(1) = 1$. Also note that $p(0)$ in the above expression is the same as the price that riskier borrowers would get at $t = 0$ in the full information case given the same value of ψ (see the last paragraph of the proof of Proposition F.2). By the same continuity argument applied in the proof of Proposition F.2, there exists an open interval of values $\psi < \tilde{p}^R$ such that $p(0)$ remains consistent with $\theta_A(0) = 1$ and $\theta_B^{C,R}(1) = 1$. ■

Proof of Proposition 2:

Proof: Lemmas M.1. and M.2 have proven existence of active equilibria for $\psi \geq (1-((1-\sigma)(\pi^R)^2/\eta)\tilde{p}$. Once ψ falls below this threshold, as the proof of Lemma M.2 suggests, the default rate at $t = 1$ would become unity if riskier agents continued to trade through the credit market. Securities sold at $t = 0$ would become worthless. All agents, however, can trade through the money market.

To prove that a stationary monetary equilibrium exists for $\psi \leq \psi_2$, first suppose that $\psi_2 \geq \alpha/beta$. In this case, over the interval $\psi \in [\alpha/beta, \psi_2]$, the following stationary monetary equilibrium can be conjectured: $\theta_A = \theta_B^{M,S} = \theta_B^{M,R} = 1$. Given these conjectured values, the money market clearing conditions are:

$$\begin{aligned} p^M(f) &= N_A M \\ p^M(s) &= N_B \eta M \end{aligned}$$

Hence,

$$\frac{p^M(f)}{p^M(s)} = \psi$$

Since $\beta/\alpha > \psi_2 \geq \psi \geq \alpha/\beta$, this rationalises the choices of θ by the various types of agents.

Over $\psi < \alpha/\beta$, the following stationary monetary equilibrium can be conjectured: $\theta_A = 1$, $\theta_B^{M,S} = \theta_B^{M,R} = \psi\beta/\alpha \equiv \theta_B^M$. In this case, the market-clearing conditions imply

$$\frac{p^M(f)}{p^M(s)} = \frac{\psi}{\theta_B^M} = \frac{\alpha}{\beta}$$

which verifies that $\theta_A = 1$ and $\theta_B^{M,i} < 1$, $i = S, R$.

Now suppose that $\psi_2 < \alpha/\beta$. In this case, a stationary equilibrium can be constructed with $\theta_A = 1$, $\theta_B^{M,S} = \theta_B^{M,R} = \psi\beta/\alpha$ along the lines of the above paragraph.

Is the monetary equilibrium robust to deviations, *i.e.*, can some subset of riskier borrowers induce lenders back into the credit market by offering security prices that dominate monetary trade for both of them? In order for lenders to expect any sort of repayment from riskier

borrowers, the stationary price of securities has to equal at least $\hat{p}^R$, as shown in Lemma 1. Hence, the return to lenders is at most $\pi^R/\hat{p}^R = 1/\tilde{p}^R$. In the money market, given the conjectured equilibrium, apple producers receive a return of $1/\psi$ per period. Over the range of values in question, $\psi < \tilde{p}^R$. Hence, the return from monetary trades exceeds anything that riskier borrowers could credibly offer to apple producers. ■

Proof of Proposition 3:

Proof: We begin by accepting the conjecture that equilibria involve no intentional default by any borrowers and that safer banana producers participate in both money and credit trades. In a stationary equilibrium, this implies

$$\beta^2 \pi^S \frac{p^M(f)}{p^M(s)} = p$$

For a complemetary willingness of apple producers to trade in both markets,

$$p = q \cdot \frac{p^M(f)}{p^M(s)}$$

Hence, it is necessary that

$$\beta^2 \pi^S = q$$

where

$$q = \frac{\pi^S \sigma \chi^S + \pi^R(1-\sigma)}{\sigma \chi^S + (1-\sigma)}$$

The above two equations can be solved for χ^S:

$$\chi^S = \frac{\beta^2 \pi^S - \pi^R}{(1-\beta^2)\pi^S} \cdot \frac{1-\sigma}{\sigma}$$

The supporting value of ξ_A can be found from the credit and money market clearing conditions for p and $p^M(f)/p^M(s)$ respectively, plus the lenders' indifference condition, stated above. This yields

$$\xi_A = \frac{\pi^R(1-\sigma) + \pi^S \chi^S \sigma}{\eta}$$

The market-clearing condition for securities implies:

$$p = \frac{\psi \eta \xi_A \theta_A}{(1-\sigma) + \chi^S \sigma}$$

which, after plugging for ξ_A and χ^S yields $p = \beta^2 \pi^S \psi \theta_A$. From the lender's indifference condition, accounting for the fact that $\beta^2 \pi^S = q$, this yields $p^M(f)/p^M(s) = \psi \theta_A$. Since $\psi \leq \beta/\alpha$, an equilibrium can be constructed in which $p^M(f)/p^M(s) = \psi$ and $\theta_A = 1$.

Given this value of $p^M(f)/p^M(s)$, safer agents who trade through money always sell one unit of their outputs. Also, since $p = \beta^3 \pi^S/\alpha > \hat{p}^S$, safer agents who trade through credit have no incentive to default. Hence, as implied by Lemma 6, neither do riskier agents. This verifies the conjectured default-free equilibrium. ■