Regulating International Capital Flows: An Externality View

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December 13, 2007

Preliminary and Incomplete – Please Do Not Quote

Abstract

This paper shows that international capital flows impose a macroeconomic externality on economies that can make them more prone to financial instability and crises. Every capital inflow entails future outflows in the form of repayments, dividends, or profit distributions. However, private recipients of capital inflows do not internalize that such outflows lead to a general macroeconomic tightening of liquidity in states of nature when international borrowing constraints are binding (such as during financial crises).

Specifically, capital outflows force a reduction in aggregate demand and put pressure on exchange rates, which in turn reduces the country's creditworthiness further, tightening borrowing constraints and leading to a vicious cycle of falling exchange rates, tightening borrowing constraints and falling aggregate demand. Therefore private recipients of capital flows impose an externality on the rest of the economy – a social planner would value payoffs in constrained states more highly than decentralized agents do.

We construct a "social pricing kernel" that prices emerging market liabilities at their true social cost and show that the externality created by a particular asset depends on the covariance of its payoffs with the extent of borrowing constraints (i.e. with financial crises). For example, uncontingent foreign currency-denominated assets, which mandate high payoffs in crisis states, are associated with large externalities; by contrast, flows of foreign direct investment, which typically yield no profit flow during crises, are free of externalities.

JEL Codes: F34, F41, E44, D62
Keywords: international capital flows, financial crises, externality

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1 Introduction

In the aftermath of a series of financial crises over the past decade, researchers have re-evaluated the benefits and costs of capital market liberalization. A growing empirical literature emphasizes the dangers of financial liberalization in terms of raising output volatility (see e.g. Prasad et al., 2003) and demonstrates that some forms of international capital flows make emerging market economies more vulnerable to sudden stops and financial crises (see e.g. Calvo et al., 2004; Levy Yeyati, 2006). This leaves the notion that controls on at least some forms of international capital flows to emerging markets might be desirable.

However, this begs the following question: if rational private agents choose to accept particular forms of international capital flows, then why should this decision not be socially efficient? Should individuals not be allowed to 'gamble'? Or, to put it more starkly, why shouldn't it be efficient for countries to experience financial crises in some (unfortunate) states of nature?

The main explanation in the existing literature has been moral hazard: firms take on risky liabilities so as to take advantage of bailout guarantees when things go wrong (see Krugman, 1998). However, risky forms of finance seem to pervasive that they could be fully accounted for by moral hazard: Even firms that were unlikely to be bailed out could be found to engage in risky forms of finance (Eichengreen and Hausmann, 1999). More generally, in many recent financial crises the sums provided by government bailouts were insufficient to cover many of the firms that went bust. This paper shares the view that firms mis-priced the social cost of repayments to creditors during crisis states. However, we differ in the cause for this mispricing: instead of government bailouts accounting for the difference between private and social cost, this paper identifies a macroeconomic externality that drives a wedge between the two.

We show that individual firms fail to internalize the effects of their borrowing decisions on the tightness of international borrowing constraints. Repayments in states of nature when borrowing constraints are triggered (such as during financial crises) necessitate a current account reversal. This depreciates the emerging market's exchange rate, which can in turn lead to contractionary effects, e.g. because depreciations reduce the dollar value of borrowers' collateral.[1]

Private recipients of capital flows do not internalize the macroeconomic implications of their behavior, in particular that their behavior in aggregate affects the tightness of individual collateral constraints. Therefore they impose an externality on the rest of the economy when receiving international capital flows. We show that a social planner would value payoffs in constrained states more highly than decentralized agents do and construct a "social pricing kernel" that prices emerging market liabilities at their true social cost. Specifically, the externality created by a particular capital inflow depends on the covariance of its payoffs with the tightness of borrowing constraints (i.e. with financial crises). For example, uncontingent dollar-denominated assets, which mandate high payoffs in crisis states, are associated with large externalities; by contrast, flows of foreign direct investment, which typically yield no profits during crises, are free of

[1] There is a long literature on contractionary depreciations in developing countries. See e.g. Frankel (2005) for an overview.

externalities.

Our paper is also related to the literature on third-generation models of currency crises (see e.g. Krugman, 1998, 1999; Chang and Velasco, 2001). As in those models, pro-cyclical fluctuations in firms' *external* borrowing capacity lie at the heart of the mechanism that is responsible for the externality. This provides further support for the importance of the externality that we identify.

Each individual agent takes the volatility of exchange rates in the economy as a given, when in fact the liability structure of the economy has an important effect on macroeconomic volatility and therefore exchange rate volatility. Higher exchange rate volatility means stronger depreciations in low output states, and correspondingly a lower dollar value of domestic collateral and tighter borrowing constraints during financial crises.

International investors only care about their private payoffs, not about the social costs that financial crises impose on emerging markets. Therefore the price that they charge does not reflect the potential costs of binding borrowing constraints. As a result of this faulty price signal, decentralized agents fail to internalize any externalities of capital flows and contract an inefficient structure and level of liabilities.

The setting in which we analyze the problem is a small open emerging market economy with three time periods, labeled 0, 1 and 2. There are two goods, tradables and non-tradables, the relative price of which represents the real exchange rate.

In period 0, small domestic agents access international capital markets to finance an investment project. In period 1, domestic agents produce their output of tradable and non-tradable goods. The former is affected by an aggregate productivity shock.[2] The exchange rate adjusts in order to equilibrate aggregate demand and supply. In case of a negative shock, for example, aggregate demand for both non-tradable and tradable goods falls; by implication the price of non-tradables has to decline (i.e. the real exchange rate depreciates) so as to restore demand for non-tradables to the given supply and clear the market. As a result, the price of non-tradables is pro-cyclical and moves in parallel with aggregate demand, i.e. the real exchange rate appreciates in high states and depreciates in low states.[3] Consequently, the repayments on uncontingent liabilities such as dollar debt (denominated in tradable goods) are high in low states and low in high output states, exacerbating the impact of aggregate shocks. By contrast, the repayments on contingent forms of liabilities such as foreign direct investment or local currency debt move in parallel with aggregate demand, mitigating the impact of aggregate shocks and therefore providing excellent insurance against consumption risk.

[2]This can be thought of e.g. as a productivity slowdown in the tradable sector, as experienced by Thailand in 1996/97, or as a devaluation of a trading partner or a competitor in export markets, which affected other East Asian countries after Thailand's devaluation and Argentina after Brazil's devaluation.

[3]We chose the given model of the real exchange rate solely for analytical convenience – as we will show below, the price of non-tradables will be linear in aggregate demand in our setup. Alternative models of the exchange rate would entail the same qualitative results, as long as the exchange rate depreciates on average in response to strong negative shocks. This condition is very likely to be satisifed in emerging markets, even if the official exchange rate regime is a peg, as demonstrated e.g. by Argentina in 2001/02.

International investors' valuation of domestic collateral depends on the exchange rate. The larger the repayments to international investors during a financial crisis, the greater the resulting current account reversal and the larger the exchange rate depreciation. By extension the stronger is the fall in the international value of domestic collateral for a given shock size. This reduces the borrowing capacity of firms that own collateral denominated in local currency, i.e. it tightens borrowing constraints and makes financial crises more frequent and more severe. The price of international capital flows reflects the social cost of it to international investors, but not the indirect effect of macroeconomic instability on collateral values during crisis states. Decentralized agents thus fail to correctly price their liabilities.

Next we identify a "social pricing kernel," which represents the true social cost of repayments across the different states of the world (analogous to a typical pricing kernel, which reflects how much a private agent values payoffs across different states of the world.) The difference between the two concepts is given by the social cost that a repayment in a given state causes because of its effect of tightening borrowing constraints. In unconstrained states of the world, private and social pricing kernels coincide. In constrained states of the world, the social planner values payoffs more than decentralized agents, and the discrepancy grows larger the tighter borrowing constraints.

The paper discusses a number of policy remedies to correct the externalities of international capital flows. One efficient way of doing so would be to impose a tax on each asset that lifts its price to the social cost as given by the social pricing kernel. In practice, it might be advisable to ban certain forms of capital flows outright.

In methodology, this paper is most closely related to the literature on the financial accelerator in macroeconomic models (see for example Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997). Building on these ideas, Krugman (1999) described financial crises as involving a debt deflation mechansim of falling exchange rates, tightening collateral constraints and falling output. Mendoza (2005) demonstrates that such a mechanism can quantitatively account for the dynamics of emerging market business cycles, i.e. for long periods of relatively smooth cycles interspersed by infrequent financial crises when financial constraints become binding. These two works take the use of uncontingent liabilities as given. Our contribution to this listerature is to show that individual agents' failure to internalize the effects of the financial accelerator during crises creates an externality that leads to greater macroeconomic instability.

The remainder of our paper is organized as follows. Section 2 introduces a benchmark model with perfect risk markets and no borrowing constraints. In section 3, we introduce borrowing constraints that depend on collateral and show the central externality result of the paper, i.e. that agents in the decentralized equilibrium do not internalize the effects of their choice of liability structure on the value of their collateral in low states. Section 4 discusses a number of policy implications. Finally, section 5 concludes.

2 Benchmark Model

2.1 Analytical Environment

In our analytical model, we analyze an emerging market represented by a small open economy. We assume that there are three time periods, labeled by $t = 0, 1, 2$, and two sets of agents: First, the emerging market economy is inhabited by a continuum of domestic agents of mass 1. Secondly, there is a continuum of international investors, who are large in comparison to the emerging market. We discuss each in detail below.

There are two perishable goods in the economy, tradable goods T and non-tradable goods N. Tradable goods can be moved costlessly across borders and can be borrowed or lent abroad. Non-tradable goods have to be consumed in the domestic economy in the period of their production. The prices of the two goods are denoted by $p_{T,t}$ and $p_{N,t}$. We choose tradables as the numeraire so that $p_{T,t} \equiv 1$. By implication, $p_{N,t}$ represents the price of non-tradables relative to tradables, which is a measure of the real exchange rate. The economy is subject to a random productivity shock that depends on the state of nature $\omega \in \Omega$, where Ω is the set of potential outcomes. The shock is realized at the beginning of period 1 and is observed by all agents.

2.2 Domestic Agents

Domestic agents are risk averse and obtain utility from consuming tradable goods $C_{T,t}$ and non-tradable goods $C_{N,t}$ according to the utility function

$$U = E\left\{\sum_{t=1}^{2} \beta^t \hat{u}(C_t)\right\} \qquad \text{where} \qquad C_t = C_{T,t}^{\sigma} C_{N,t}^{1-\sigma}$$

E is the expectations operator, β represents agents' discount factor, and $\hat{u}(\cdot)$ is their period utility function, which satisfies $\hat{u}'' < 0 < \hat{u}'$. C_t is an aggregator of tradable and non-tradable consumption, where the parameters σ and $1-\sigma$ are the expenditure shares of tradables and non-tradables in agents' optimal consumption bundle.[4]

We assume that agents start out with an initial amount of wealth (or debt, for negative values) of W_0. Agents need to invest a fixed amount of tradables $\bar{I}$ in both periods 0 and 1.[5] As a return on their investment, they produce a bundle of tradable and non-tradable goods of $(Y_{T,t}^{\omega}, \bar{Y}_N)$. The fixed amounts of production capture that production factors cannot be re-allocated instantaneously. Instead, relative prices have to adjust, i.e. the real exchange rate appreciates or depreciates.

[4]The Cobb-Douglas form in which tradable and non-tradable goods enter utility implies that the real exchange rate is a linear function of the output shock. This in turn guarantees that the two assets, uncontingent debt and contingent equity, span the entire state space and therefore that risk markets are complete for domestic agents. While our externality result holds as long as both tradables and non-tradables are normal goods, the chosen specification allows us to derive the result analytically.

[5]Endogenizing the amount of investment would not change the basic results – agents would equalize the expected marginal product of investment to the cost of capital on international markets. In the presence of binding borrowing constraints, agents would reduce not only current consumption, but also investment, and would thereby have negative effects on future output. This could potentially increase the size of the externality even further.

The output of tradables $Y^{\omega}_{T,1}$ is subject to the aggregate productivity shock ω in period 1. This shock can stem e.g. from a general slowdown in productivity growth in that sector or from an adverse terms-of-trade shock, such as a neighboring country's devaluation or a fall in the world market price of the country's main exports. The expected value of first period output equals $E[Y^{\omega}_{T,1}] = \bar{Y}_T$. For simplicity, suppose that tradable production in period 2 is constant and also equals to $Y^{\omega}_{T,2} = \bar{Y}_T$.

In the given representative agent framework, there is no scope for a domestic credit market, since agents are all identical and have a common, stricly concave utility function. They are either all lenders or all borrowers. However, agents can obtain capital from abroad: they have a choice of uncontingent debt F_t or contingent debt (equity) L_t. Both assets are normalized to yield one unit of tradables in period $t-1$ from abroad.[6] However, they differ in the amounts to be repaid in period t. Uncontingent debt F_t mandates the repayment of an unconditional return of $R^* F_t$ tradables in period t, where R^* is determined on world markets. The return $R^{\omega}_{L,t}$ on contingent debt/equity is indexed to the real exchange rate, i.e. is proportional to the price of non-tradables: $R^{\omega}_{L,t} \cong p^{\omega}_{N,t}$. Taking on an amount of L_t of equity in period $t-1$ thus entails an obligation to repay $R^{\omega}_{L,t} L_t$ in period t, where $R^{\omega}_{L,t}$ depends on the realization of the real exchange rate.[7]

In our benchmark model, we assume that there are no capital market imperfections, and that agents can take on their optimal amounts of debt and equity while maintaining strictly positive consumption in all states of the world, i.e. there is no bankruptcy. In the following section, we will introduce borrowing constraints that limit the amount of finance that agents can take on depending on the state of their balance sheets.

We can summarize domestic agents' optimization program in the following form:

$$\max_{\{C^{\omega}_t, C^{\omega}_{T,t}, C^{\omega}_{N,t}, F^{\omega}_t, L^{\omega}_t\}_{t=1,2}} E\left\{\sum_{t=1}^{2} \beta^t \hat{u}(C^{\omega}_t)\right\} \tag{1}$$

$$\text{s.t. } C^{\omega}_t = [C^{\omega}_{T,t}]^{\sigma} [C^{\omega}_{N,t}]^{1-\sigma} \tag{2}$$

$$\bar{I} \leq W_0 + F_1 + L_1 \tag{3}$$

$$C^{\omega}_{T,1} + p^{\omega}_{N,1} C^{\omega}_{N,1} + \bar{I} = Y^{\omega}_{T,1} + p^{\omega}_{N,1} \bar{Y}_N - R^* F_1 - R^{\omega}_{L,1} L_1 + F^{\omega}_2 + L^{\omega}_2 \tag{4}$$

$$C^{\omega}_{T,2} + p^{\omega}_{N,2} C^{\omega}_{N,2} = \bar{Y}_T + p^{\omega}_{N,2} \bar{Y}_N - R^* F^{\omega}_2 - R^{\omega}_{L,2} L^{\omega}_2$$

2.3 International Investors

International capital markets are populated by a continuum of identical competitive investors. We assume that international investors are large in comparison to the small

[6] Naturally, foreigners can only provide domestic agents with tradable goods, since non-tradables cannot be moved across borders.

[7] The contingent asset can be interpreted both as local currency-denominated debt and as equity. As we will show below, our specification of the contingent asset guarantees that risk markets are complete, even in the presence of borrowing constraints. This would not be the case if the contingent asset was directly dependent on the output shock for tradables $Y^{\omega}_{T,1}$. However, we will discuss the welfare effects of other contingent assets below.

emerging market economy, so that the equilibrium in international capital markets is exogenous to events in the emerging market economy.

As a result, investors' pricing kernel M_t^ω is given exogenously.[8] In equilibrium, the returns $R_{i,t}^\omega$ on any asset i in which international investors can invest have to satisfy the standard pricing condition

$$E\left\{R_{i,t}^\omega M_t^\omega\right\} = 1 \tag{5}$$

This pins down the risk-free return that international investors demand on uncontingent debt as $R^* = 1/E[M_t^\omega]$, which we assume constant over time.

2.4 Definition of Equilibrium

We take the equilibrium in international capital markets as given, since we assumed that the emerging market economy is small compared to international markets. Given international investors' pricing kernel M_t^ω and risk free return R^*, an equilibrium in the emerging market economy is defined as

- an allocation $(C_t^\omega, C_{T,t}^\omega, C_{N,t}^\omega, F_t^\omega, L_t^\omega)$ and
- prices and returns $(p_{N,t}^\omega, R_{L,t}^\omega)$ for $t = 1, 2$ and all $\omega \in \Omega$
- which are consistent with international investors' pricing condition (5)
- which satisfy domestic agents' optimization problem (1) given (2)
- and which clear goods markets at all times $t = 0, 1, 2$ and in all states $\omega \in \Omega$, i.e.

$$\begin{aligned}
\text{for non-tradables:} \qquad C_{N,t}^\omega &= \bar{Y}_N \\
\text{for tradables:} \qquad \bar{I} &= W_0 + F_1 + L_1 \\
C_{T,1}^\omega + \bar{I} &= Y_{T,1}^\omega - R^* F_1 - R_{L,1}^\omega L_1 + F_2^\omega + L_2^\omega \\
C_{T,2}^\omega &= \bar{Y}_T - R^* F_2^\omega - R_{L,2}^\omega L_2^\omega
\end{aligned}$$

2.5 Determination of Investors' Equilibrium Returns

The emerging market economy under study is small, and so international investors' supply of F_1 and F_2^ω is horizontal at R^* for domestic agents, up to the point where borrowing constraints start to bind.Since there is no uncertainty in period 2, one of the two assets is redundant that period. We set w.l.o.g. $L_2^\omega = 0$.

The price at which international investors are willing to supply equity L_1 in the first period can be determined from investors' equilibrium pricing condition $E[R_{L,1}^\omega M_1^\omega] = 1$. Let us define ρ as a measure of the equity premium so that

$$(1 - \rho)E[R_{L,1}^\omega] = R^*$$

[8]For an appropriate definition of M_1^ω, most asset pricing models can be captured by the following pricing condition. Other common terms used for M_t^ω in the asset pricing literature are intertemporal marginal rate of substitution or stochastic discount factor.

As discussed above the payoff $R^{\omega}_{L,1}$ is linear in the real exchange rate, and our definition of ρ implies that it is in expectation $\frac{R^*}{1-\rho}$. According to simple no-arbitrage considerations we can denote the state contingent return on equity as

$$R^{\omega}_{L,1} = \frac{p^{\omega}_{N,1}}{E[p^{\omega}_{N,1}]} \cdot \frac{R^*}{1-\rho} \tag{6}$$

Substituting this expression into investors' equilibrium pricing condition $E[R^{\omega}_{L,1} M^{\omega}_1] = 1$, we find that

$$\rho = -R^* \mathrm{Cov}\left(\frac{p^{\omega}_{N,1}}{E[p^{\omega}_{N,1}]}, M^{\omega}_1\right) = -\mathrm{Cov}\left(\frac{p^{\omega}_{N,1}}{E[p^{\omega}_{N,1}]}, \frac{M^{\omega}_1}{E[M^{\omega}_1]}\right) \tag{7}$$

This capture the standard result that international investors do not mind holding risk that is uncorrelated to their pricing kernel M^{ω}_1. However, they do require a higher return for holding risk that is negatively correlated with their pricing kernel M^{ω}_1.

If investors were risk-neutral, i.e. the pricing kernel M^{ω}_1 was constant, or if the real exchange rate was uncorrelated to M^{ω}_1, then $\rho = 0$ and investors would supply any finite amount of local currency debt at actuarially fair prices. On the other hand, if investors are risk-averse, i.e. the pricing kernel is a non-degenerate random variable, and if the real exchange rate $p^{\omega}_{N,1}$ is negatively correlated with the pricing kernel, i.e. on average depreciated when investors value consumption highly and appreciated when they value consumption less, then investors demand a positive risk premium $\rho > 0$. For the rest of our analysis we thus assume that investors' pricing kernel is such that $\rho > 0$.

2.6 Domestic Agents in Competitive Equilibrium

In competitive equilibrium, representative agents in the emerging market economy solve maximization problem (1). We can substitute the consumption aggregator C^{ω}_t, the constraint for tradable consumption $C^{\omega}_{T,t}$ and for first period uncontingent borrowing F_1 to arrive at the following maximization problem:

$$\max_{C^{\omega}_{N,t}, L_1, F^{\omega}_2} E\Big\{\hat{u}\left([Y^{\omega}_{T,1} + p^{\omega}_{N,1}(\bar{Y}_N - C^{\omega}_{N,1}) - \bar{I} + R^*(W_0 - \bar{I}) + L_1\left(R^* - R^{\omega}_{L,1}\right) + F^{\omega}_2]^{\sigma}[C^{\omega}_{N,1}]^{1-\sigma}\right)$$
$$+ \beta\hat{u}\left([\bar{Y}_T + p^{\omega}_{N,2}(\bar{Y}_N - C^{\omega}_{N,2}) - R^* F^{\omega}_2]^{\sigma}[C^{\omega}_{N,2}]^{1-\sigma}\right)\Big\} \tag{8}$$

In the decentralized equilibrium, individual agents take the aggregate price level $p^{\omega}_{N,t}$ of non-tradables as well as the returns R^* and $R^{\omega}_{L,1}$ as given. We obtain the following first-order conditions:

$$\begin{aligned}
\mathrm{FOC}(C^{\omega}_{N,t}):\ & \hat{u}'(C^{\omega}_t)\left(\sigma[C^{\omega}_{T,t}]^{\sigma-1}(-p^{\omega}_{N,1})[C^{\omega}_{N,t}]^{1-\sigma} + [C^{\omega}_{T,t}]^{\sigma}(1-\sigma)[C^{\omega}_{N,t}]^{-\sigma}\right) = 0 \\
\mathrm{FOC}(L_1):\ & E\left\{\hat{u}'(C^{\omega}_1)\sigma[C^{\omega}_{T,1}]^{\sigma-1}[C^{\omega}_{N,1}]^{1-\sigma}\left[R^* - R^{\omega}_{L,1}\right]\right\} = 0 \\
\mathrm{FOC}(F^{\omega}_2):\ & \hat{u}'(C^{\omega}_1)\sigma[C^{\omega}_{T,1}]^{\sigma-1}[C^{\omega}_{N,1}]^{1-\sigma} = \beta R^* \hat{u}'(C^{\omega}_2)\sigma[C^{\omega}_{T,2}]^{\sigma-1}[C^{\omega}_{N,2}]^{1-\sigma}
\end{aligned}$$

The first expression relates the consumption of non-tradables and tradables in each period. The second condition equalizes the expected marginal utility provided by equity L_1 to its marginal cost. The third condition describes the intertemporal tradeoff between first- and second-period consumption.

We employ a recursive solution strategy: First, we describe the determination of the exchange rate determination implied by FOC($C^{\omega}_{N,t}$) and use FOC(F^{ω}_2) to solve for the equilibrium in periods 1 and 2, given a choice of liability structure as expressed by L_1 in period 0. Then, we use this solution to analyze agents' optimal choice of liability structure in period 0.

First Order Condition on $C^{\omega}_{N,t}$: Determination of the Real Exchange Rate

The first order condition on $C^{\omega}_{N,t}$ determines equilibrium in the market for non-tradable goods. In our setup, the domestic supply of non-tradable goods is fixed at $\bar{Y}_N$, a modeling assumption that reflects that production takes time and output cannot be adjusted instantaneously. Market clearing for non-tradable goods thus always requires that demand equals the given supply, i.e. $C^{\omega}_{N,t} = \bar{Y}_N$. Changes in market conditions thus affect prices rather than quantities in the non-tradable sector. Tradable goods, on the other hand, can be imported or exported if agents borrow or lend abroad.

If we substitute the market clearing condition for non-tradable goods into the first order condition on $C^{\omega}_{N,t}$, we find

$$p^{\omega}_{N,t} = \frac{1-\sigma}{\sigma} \cdot \frac{C^{\omega}_{T,t}}{\bar{Y}_N} = \varsigma \cdot \frac{C^{\omega}_{T,t}}{\bar{Y}_N} \tag{9}$$

In equilibrium the real exchange rate $p^{\omega}_{N,t}$ is proportional to the economy's absorption of tradables $C^{\omega}_{T,t}$.[9] (To save on notation we define $\varsigma = \frac{1-\sigma}{\sigma}$.)

If e.g. a negative shock to agents' production of tradables $Y^{\omega}_{T,1}$ occurs, tradable goods become scarcer and non-tradable goods relatively more abundant. For equilibrium to be restored, the relative price of tradables has to rise, i.e. $p^{\omega}_{N,1}$ falls – the real exchange rate depreciates. Similarly, if tightening external borrowing constraints force agents to reduce their borrowing, they have to cut back on their tradable consumption $C^{\omega}_{T,1}$ and export more (or import fewer) tradables, whereas the supply of non-tradable goods remains constant (non-tradable production cannot be shipped overseas) and the same quantity has to be consumed. Equilibrium again requires that the relative price of non-tradables $p^{\omega}_{N,1}$ falls.

We can apply condition (9) that the real exchange rate is linear in tradable consumption to expressions (6) and (7) for the return on equity/local currency debt $R^{\omega}_{L,1}$

[9] The simple linear relationship stems from the Cobb-Douglas aggregator (2). For other utility functions, we would obtain a non-linear but still monotonic relationship between $p^{\omega}_{N,t}$ and $C^{\omega}_{T,t}$, so long as both tradable and non-tradable goods are normal. The real depreciation mechanism and the externality result of this paper are thus robust to any standard utility function. However, if (9) is non-linear and there are more than two shocks, risk markets would no longer be complete. This creates an additional externality that would warrant exchange rate intervention by the social planner. We will touch upon this issue in the following subsections, but we leave a more detailed analysis to future research.

and the risk premium ρ to obtain

$$R^{\omega}_{L,1} = \frac{R^*}{1-\rho} \cdot \frac{C^{\omega}_{T,1}}{E[C^{\omega}_{T,1}]} \tag{10}$$

$$\rho = -R^* \mathrm{Cov}\left(\frac{C^{\omega}_{T,1}}{E[C^{\omega}_{T,1}]}, M^{\omega}_1\right) \tag{11}$$

The first expression captures that the payoff of equity is a linear function of domestic agents' consumption of tradable goods. This illustrates that it provides excellent insurance against consumption risk: domestic agents have to pay back more in precisely the states that their consumption is high, and pay back less when their consumption is low.

The second equation expresses investors' risk premium as a function of the covariance of domestic tradable consumption, which drives the exchange rate, with investors' pricing kernel M^{ω}_1. The more domestic tradable consumption is negatively correlated with investors' risk factors, the higher the risk premium that investors demand as a compensation for taking on the risky asset.

In our further analysis below, we will be able to greatly save on notation by employing two transformations: Firstly, we substitute the utility function $\hat{u}(\cdot)$ by the following function:

$$u(C_T) = \hat{u}(C^{\sigma}_T C^{1-\sigma}_N) = \hat{u}(C^{\sigma}_T \bar{Y}^{1-\sigma}_N) \tag{12}$$

where we take advantage of the fixed supply and the market clearing condition for non-tradables $C^{\omega}_{N,t} = \bar{Y}_N$. Secondly, we employ the transformation

$$N = L_1 \cdot \frac{R^*}{(1-\rho)E[C^{\omega}_{T,1}]}$$

L_1 measures the amount of local currency debt/equity in units of tradable goods. Both instruments provide insurance against consumption risk, so we can think of N roughly as the amount of iNsurance that agents take on relative to their expected level of consumption.

In conjunction with condition (10) for the return on insurance this simplifies expression (4) for agents' first-period consumption to

$$C^{\omega}_{T,1} = Y^{\omega}_{T,1} - (1+R^*)\bar{I} + R^* W_0 - N\left[C^{\omega}_{T,1} - (1-\rho)E[C^{\omega}_{T,1}]\right] + F^{\omega}_2 \tag{13}$$

This formulation illustrates that, for a given amount of total borrowing, raising N, i.e. increasing the amount of insurance at the expense of uncontingent debt, is equivalent to swapping the risky consumption stream $C^{\omega}_{T,1}$ against investors' certainty equivalent $(1-\rho)E[C^{\omega}_{T,1}]$.

First Order Condition on F^{ω}_2: Euler Equation

The first order condition on F^{ω}_2 is the Euler equation that determines agents' optimal intertemporal allocation across periods 1 and 2. Using the transformed utility function from above, it can be represented as

$$\mathrm{FOC}(F^{\omega}_2)\colon\ u'(C^{\omega}_{T,1}) = \beta R^* u'(C^{\omega}_{T,2})$$

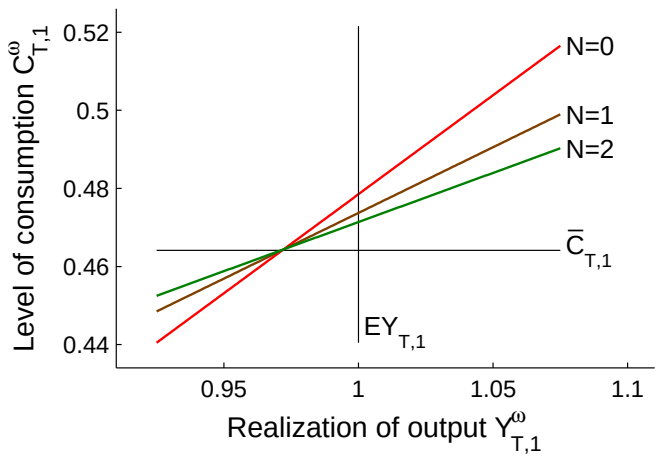

Figure 1: The figure depicts the level of consumption $C^{\omega}_{T,1}$ as a function of the output shock $Y^{\omega}_{T,1}$ for several different levels of insurance N. As can be seen, more insurance makes consumption less sensitive to the shock: agents can trade in claims on their variable level of consumption $C^{\omega}_{T,1}$ against investors' certainty equivalent $\bar{C}_{T,1} = (1-\rho)E[C^{\omega}_{T,1}]$, represented by the horizontal line. This 'rotates' consumption around $\bar{C}_{T,1}$. The vertical line represents the average level of output $\bar{Y}_T$.

Making the standard assumption that $\beta R^* = 1$, this condition implies that $u'(C^{\omega}_{T,1}) = u'(C^{\omega}_{T,2})$, i.e. domestic agents set F^{ω}_2 in every state such as to smooth out the shock and perfectly equalize consumption over the two periods:

$$C^{\omega}_{T,1} = C^{\omega}_{T,2} = \bar{Y}_T - R^* F^{\omega}_2$$

This results in an expression for the optimal amount of borrowing F^{ω}_2 of

$$F^{\omega}_2 = \frac{\bar{Y}_T - C^{\omega}_{T,1}}{R^*} \tag{14}$$

Let us substitute this finding into agents' level of consumption (13) and solve for

$$C^{\omega}_{T,1} = \frac{\bar{Y}_T + R^* Y^{\omega}_{T,1} - R^*(1+R^*)\bar{I} + (R^*)^2 W_0 + (1-\rho)NR^* E[C^{\omega}_{T,1}]}{1 + R^*(1+N)} \tag{15}$$

This equation indicates how tradable consumption depends on the productivity shock $Y^{\omega}_{T,1}$ for a given level of insurance N. It demonstrates that consumption is less sensitive to productivity shocks the more local currency debt domestic agents have taken on. This is also illustrated in figure 1.[10] The higher N, the flatter the lines in the figure, i.e. the less consumption varies in response to the productivity shock $Y^{\omega}_{T,1}$. In addition, note that insurance is not free, and the payment of the risk premium reduces average

[10] We used the following parameter values for this figure: We chose $Y^{\omega}_{T,1} \sim N(\bar{Y}, s^2_Y)$ with $\bar{Y}_T = \bar{Y}_N = 1$, $s_Y = .05$, $\sigma = .40$, $R^* = 1.03$, $\bar{I} = .25$ and $W_0 = -.50$. M^{ω}_1 was calibrated to be perfectly negatively correlated with $Y^{\omega}_{T,1}$ and yield a risk premium of $\rho = 3\%$ when all borrowing takes place in foreign currency.

consumption $E[C^{\omega}_{T,1}]$. Taking expectations of equation (15), we can solve for agents' average level of consumption

$$E[C^{\omega}_{T,1}] = \frac{(1+R^*)\bar{Y}_T - R^*(1+R^*)\bar{I} + (R^*)^2 W_0}{1+R^*(1+\rho N)} \tag{16}$$

The figure also illustrates another feature: from decentralized agents' perspective, taking on more insurance is equivalent to swapping the random payoff $C^{\omega}_{T,1}$ against investors' certainty equivalent $(1-\rho)E[C^{\omega}_{T,1}]$. However, in general equilibrium the variables $C^{\omega}_{T,1}$, ρ, and $E[C^{\omega}_{T,1}]$ are all endogenous. Taking the derivative of expression (15) with respect to N, we find that

$$\frac{dC^{\omega}_{T,1}}{dN} = -\frac{R^*}{1+R^*(1+N)}\left[C^{\omega}_{T,1} - \bar{C}_{T,1}\right]$$

where

$$\bar{C}_{T,1} = \frac{d(1-\rho)NE[C^{\omega}_{T,1}]}{dN} = \; = (1-\rho)E[C^{\omega}_{T,1}] - NE[C^{\omega}_{T,1}]\frac{d\rho}{dN} + (1-\rho)N\frac{dE[C^{\omega}_{T,1}]}{dN} = (1-\rho)E[C^{\omega}_{T,1}]$$

Increasing the amount of insurance N reduces decentralized agents' consumption in every state ω by the random payoff $C^{\omega}_{T,1}$ and replaces this by the constant payoff $\bar{C}_{T,1}$, which can be interpreted as the fixed increase in consumption that results in exchange for increasing the amount of insurance N and giving up some of the random $C^{\omega}_{T,1}$.

$\bar{C}_{T,1}$ consists of three parts: expected consumption discounted by the risk premium, the reduction in the risk premium for the pre-existing amount of insurance N, and the reduction in the payoff of the pre-existing N units of insurance that result from increasing N. Decentralized agents internalize the first term, i.e. they are aware that investors charge a risk premium for taking on the risky component of $C^{\omega}_{T,1}$. However, individual agents do not internalize that their aggregate behavior affects the risk premium and the payoff of the existing units of insurance. In our benchmark model, it turns out that these two terms cancel out. Intuitively, the reason for this is that increasing N does not affect the relative costs and payoffs of the existing insurance arrangement: investors do not care about whether they provide a defined contingent payoff through a small number of strongly fluctuating assets or a large number of mildly fluctuating assets. Mathematical details are provided in appendix B.

In the figure, the result is that borrowers' consumption seems to rotate around the point where $C^{\omega}_{T,1} = \bar{C}_{T,1}$ when the amount of insurance N is raised: local currency debt increases income and consumption in low states with $C^{\omega}_{T,1} < \bar{C}_{T,1}$ and reduces income and consumption in high states with $C^{\omega}_{T,1} > \bar{C}_{T,1}$.

The following proposition summarizes the implications of more insurance N for the variance of consumption and the real exchange rate as well as for the country's risk premium ρ:

Proposition 1 *The more of its debt a country denominates in contingent forms, the lower the volatility of aggregate demand, consumption and the real exchange rate; as a result, the lower is the risk premium that international investors' charge on contingent assets.*

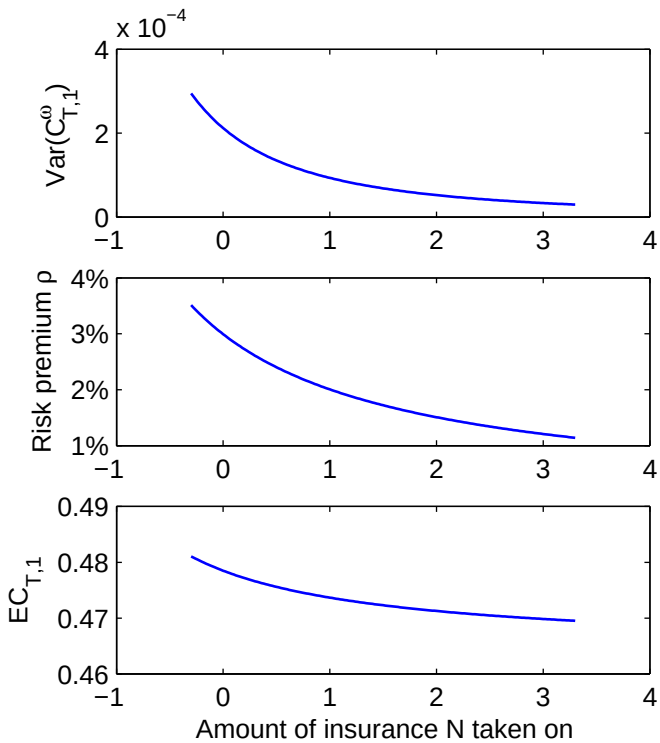

Figure 2: The first pane of the figure depicts that the variance of consumption (and of the real exchange rate) is a decreasing function of the amount of insurance N. The second pane shows the dependence of the risk premium on the level of insurance N: as more insurance makes the country less risky, the risk premium falls. The third pane illustrates the fall in average consumption as more insurance is bought. This stems from the fact that insurance is costly – borrowers need to pay a risk premium on local currency debt.

Proof. Using expression (15) for $C^{\omega}_{T,1}$, we express the standard deviation of tradable consumption as

$$\mathrm{Std}(C^{\omega}_{T,1}) = \frac{R^*}{1 + R^*(1+N)} \cdot \mathrm{Std}(Y^{\omega}_{T,1}) \tag{17}$$

This expression unambiguously decreases the higher the fraction of contingent assets, or insurance:

$$\frac{d\mathrm{Std}(C^{\omega}_{T,1})}{dN} = -\left(\frac{R^*}{1 + R^*(1+N)}\right)^2 \cdot \mathrm{Std}(Y^{\omega}_{T,1})$$

Since the real exchange rate $p^{\omega}_{N,1} = \frac{\varsigma}{Y_N} \cdot C^{\omega}_{T,1}$ is linear in tradable consumption according to (9), the same results apply to the real exchange rate and the risk premium. ■

We have illustrated the functional relationship between the level of insurance N and (i) the variance of consumption $\mathrm{Var}(C^{\omega}_{T,1})$, (ii) the risk premium ρ, and (iii) average consumption $E[C^{\omega}_{T,1}]$ in figure 2.

First Order Condition on L_1: Choice of Liability Structure

Let us next focus on agents' decision regarding how much contingent/uncontingent liabilities to take on, i.e. on how much to pay for insurance. We substitute the return on the contingent asset $R^{\omega}_{L,1} = \frac{R^*}{1-\rho} \cdot \frac{C^{\omega}_{T,1}}{E[C^{\omega}_{T,1}]}$ from condition (10) into the first order condition on L_1 and, using our transformed utility function, we obtain that

$$\text{FOC}(L_1)\text{: } E\left\{u'(C^{\omega}_{T,1})\left[p^{\omega}_{N,1} - (1-\rho)E[p^{\omega}_{N,1}]\right]\right\} = 0$$

The product $E[u'(C^{\omega}_{T,1})X^{\omega}]$ represents borrowers' valuation of an asset with payoffs X^{ω} in every state ω. The condition above thus states that borrowers' valuation of one unit of the contingent asset (payoff $p^{\omega}_{N,1}$) has to equal the valuation of its certainty equivalent, i.e. of the expected value $E[p^{\omega}_{N,1}]$ discounted by investors' risk premium ρ. Using the fact that for any two random variables X^{ω} and Y^{ω}: $E[X^{\omega}Y^{\omega}] = \text{Cov}(X^{\omega}, Y^{\omega}) + E[X^{\omega}]E[Y^{\omega}]$, this yields that

$$\text{Cov}\left\{\frac{u'(C^{\omega}_{T,1})}{E[u'(C^{\omega}_{T,1})]}, \frac{p^{\omega}_{N,1}}{E[p^{\omega}_{N,1}]}\right\} = -\rho$$

Domestic agents hold contingent assets up to the point where the additional insurance effect per unit equals the cost of obtaining the insurance, which is the risk premium. The higher this premium, the less insurance agents thus take on. On the other hand, in the extreme case that $\rho = 0$, decentralized agents would perfectly insure against consumption risk. As a result consumption would be constant and the covariance between the two variables would turn zero.

As we discussed above in equation (9), the real exchange in our model is linear in tradable consumption. This implies that

$$\frac{p^{\omega}_{N,1}}{E[p^{\omega}_{N,1}]} = \frac{C^{\omega}_{T,1}}{E[C^{\omega}_{T,1}]}$$

The covariance expression above then entails that agents insure to the point where the absolute value of the covariance between agents' marginal utility and their consumption equals the risk premium ρ. Approximating the utility function by a quadratic function[11] implies a marginal utility of $u'(C_T) = \Gamma - C_T$. This yields the condition

$$\text{Var}(C^{\omega}_{T,1}) = \rho E(C^{\omega}_{T,1})E\left[u'(C^{\omega}_{T,1})\right] \tag{18}$$

Domestic agents' optimal variance of tradable consumption is directly proportional to the risk premium, i.e. to the cost of insuring against volatility. In the case that $\rho = 0$, the optimal variance of tradable consumption would be zero, i.e. domestic agents would

[11] We assume that $u(C_T)$ is quadratic in tradable consumption, i.e. that $u(C_T) = -\frac{1}{2}(\Gamma - C_T)^2$, where Γ is a constant such that $C_T < \Gamma$. This implies that the original utility function $\hat{u}(C)$ has to take the form $\hat{u}(C) = -\frac{1}{2}\left[\Gamma - \left(\frac{C}{\bar{Y}_N^{1-\sigma}}\right)^{\frac{1}{\sigma}}\right]^2$, where C has to fulfill $\Gamma^{\sigma} > \frac{C}{\bar{Y}_N^{1-\sigma}} > \left(\frac{1-\sigma}{2-\sigma}\Gamma\right)^{\sigma}$. The two inequalities guarantee that the original utility function $\hat{u}(C)$ satisfies the standard neoclassical conditions $\hat{u}'(C) > 0 > \hat{u}''(C)$.

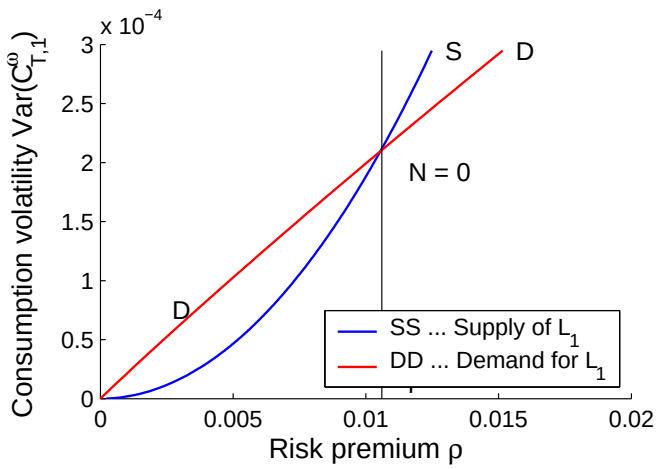

Figure 3: The figure depicts the supply locus SS and demand locus DD for insurance. Equilibrium is determined by the intersection of the two, here indicated by the vertical line ρ^* and the resulting equilibrium value of N.

insure perfectly against consumption fluctuations. In our analysis we assumed that investors charge a positive risk premium $\rho > 0$. This implies that domestic agents' insure imperfectly. They opt for higher consumption volatility the higher the risk premium.

Solution to the Decentralized Equilibrium

The solution to the decentralized equilibrium is given by the 4 equations (11), (13), (14) and (18) in the 4 variables $(\rho, N, F_2^\omega, C_{T,1}^\omega)$, where the last two equations and the last two variables are state-contingent. Conditions (3) and (9) then yield F_1 and $p_{N,1}^\omega$, and from the market clearing condition for non-tradables we trivially obtain $C_{N,t}^\omega = \bar{Y}_N$.

We can collapse this system of equations into a system of two equations in two variables, ρ and $\mathrm{Var}(C_{T,1}^\omega)$. The first equation is borrowers' optimality condition, or demand locus (18). In figure 3, this is the DD locus. It is slightly concave because of the term $E[C_{T,1}^\omega]^\omega(\Gamma - E[C_{T,1}^\omega])$, which is an inverted parabola.

The second equation can be derived from investors' supply condition through the following steps. Using condition (15), we find that

$$\mathrm{Cov}(C_{T,1}^\omega, M_1^\omega) = \frac{R^*}{1 + R^*(1+N)} \cdot \mathrm{Cov}(Y_{T,1}^\omega, M_1^\omega)$$

We have demonstrated above in (17) that a similar relationship holds for the standard deviation of consumption, i.e. $\mathrm{Std}(C_{T,1}^\omega) = \frac{R^*}{1+R^*(1+N)}\mathrm{Std}(Y_{T,1}^\omega)$. Putting the two together, it follows that

$$\rho E[C_{T,1}^\omega] = -R^*\mathrm{Cov}(C_{T,1}^\omega, M_1^\omega) = -R^* \cdot \frac{\mathrm{Cov}(Y_{T,1}^\omega, M_1^\omega)}{\mathrm{Std}(Y_{T,1}^\omega)} \cdot \mathrm{Std}(C_{T,1}^\omega) = -\xi \cdot \mathrm{Std}(C_{T,1}^\omega) \quad (19)$$

for an appropriately defined constant ξ, where we use the observation that the covariance and standard deviation of $Y_{T,1}^\omega$ are given. The risk premium that investors require

is thus linear in the standard deviation of consumption: the more consumption and the real exchange rate fluctuate, the higher the premium that international investors require to hold contingent assets. We can consider this as international investors' optimality condition or supply locus of insurance. In figure 3, this relationship is depicted as SS. The locus SS is convex since the axis in the figure depicts the variance $\text{Var}(C^{\omega}_{T,1})$, i.e. the square of the standard deviation.

Note that the amount of insurance (as represented by N) increases as we move from the top right of the figure to the bottom left along both the SS and DD loci, as the insurance asset reduces volatility in the economy. In the limit, i.e. as $N \to \infty$, both loci end up in the origin. The concavity and convexity of the two schedules guarantee a unique non-degenerate equilibrium, as indicated in the figure.

2.7 Social Planner

We analyze two different versions of the social planner's problem here. Firstly, we describe a constrained social planner who can determine the portfolio structure of decentralized agents, but who leaves the decision regarding how much to consume and borrow to them. This setup is of particular interest for policymakers who have to regulate international capital flows. In the next subsection, we will describe an all-powerful social planner who coordinates all portfolio and consumption allocations of agents.

The constrained social planner in the emerging market economy only has the capacity to determine agents' portfolio structure. Contrary to agents in the decentralized equilibrium, she internalizes the effects of her decisions on the volatility of the real exchange rate $p^{\omega}_{N,t}$. This has two equilibrium effects: First, a lower variance implies that the covariance of the real exchange rate with investors' stochastic discount factor falls, which reduces the risk premium ρ that investors charge, as described in condition (11). Secondly, as the volatility in the real exchange rate falls, each unit of the contingent asset provides less insurance, since it co-varies less with the state of the economy. To obtain a given amount of insurance, it is thus necessary to hold more of the contingent asset.

The constrained social planner leaves the decision regarding how much to consume and how much to save to the decentralized agents. In the given setup, the amount of borrowing in period 0 is pre-determined by $\bar{I}$. Decentralized agents only have a choice over how much to consume and borrow/save in period 1, i.e. $C^{\omega}_{T,1}$ versus F^{ω}_2. Given the social planner's decision regarding L_1 and the realization of the productivity shock $Y^{\omega}_{T,1}$, private agents' optimization problem in period 1 is

$$\max_{C^{\omega}_{T,1}, F^{\omega}_2} E\left\{u(C^{\omega}_{T,1}) + \beta u(\bar{Y}_T - R^* F^{\omega}_2)\right\}$$
$$\text{s.t. } C^{\omega}_{T,1} = Y^{\omega}_{T,1} - (1+R^*)\bar{I} + R^* W_0 + L_1(R^* - R^{\omega}_{L,1}) + F^{\omega}_2$$

where we have substituted $C^{\omega}_{N,t} = \bar{Y}_N$ and employed the transformed utility function $u(\cdot)$ as in (12). We can substitute the constraint into the first period utility function and derive with respect to F^{ω}_2 to find agents' Euler equation

$$\text{FOC}(F^{\omega}_2) : u'(C^{\omega}_{T,1}) = \beta R^* u'(C^{\omega}_{T,2})$$

They perfectly smooth their consumption over time. As in the decentralized equilibrium, this results in a level of second-period borrowing of

$$F_2^\omega = \frac{\bar{Y}_T - C_{T,1}^\omega}{R^*} \tag{20}$$

Taking this into account, we can formulate the constrained social planner's maximization problem as

$$\begin{aligned}
\max_{C_{T,1}^\omega, F_2^\omega, N, \rho, EC_{T,1}} \; & E\left\{u(C_{T,1}^\omega) + \beta u(\bar{Y}_T - R^* F_2^\omega)\right\} \qquad (21)\\
\text{s.t. } C_{T,1}^\omega &= Y_{T,1}^\omega - \bar{I} - R^*(\bar{I} - W_0) - N[C_{T,1}^\omega - (1-\rho)EC_{T,1}] + F_2^\omega \\
F_2^\omega &= \frac{\bar{Y}_T - C_{T,1}^\omega}{R^*} \\
\rho EC_{T,1} &= -R^* \mathrm{Cov}(C_{T,1}^\omega, M_1^\omega) = -E\left[(C_{T,1}^\omega - EC_{T,1})(R^* M_1^\omega - 1)\right] \\
EC_{T,1} &= E[C_{T,1}^\omega]
\end{aligned}$$

Here we have used the transformation $N = L_1 \cdot \frac{R^*}{(1-\rho)E[C_{T,1}^\omega]}$ and substituted for the real exchange rate $p_{N,1}^\omega = \frac{\varsigma}{Y_N} C_{T,1}^\omega$ according to equation (9). Since $u(C_{T,1}^\omega) = u(C_{T,2}^\omega)$ and the constraint on F_2^ω always has to hold with equality, we can simplify the constrained social planner's objective to $(1+\beta)u(C_{T,1}^\omega)$ or, dropping the constant, $u(C_{T,1}^\omega)$.

Note that we use the variable $EC_{T,1}$ as a separate argument in the maximization problem, which is linked to expected consumption $E[C_{T,1}^\omega]$ in the last constraint. This is necessary, since the expectations around the social planner's objective and for expected consumption in the covariance condition need to be calculated using two different integrating variables. This treatment leads to the following Lagrangian for the optimization problem:

$$\begin{aligned}
\mathcal{L}^{SP} = E\Bigg\{ & u\left(C_{T,1}^\omega\right) - \nu\left[\rho EC_{T,1} + (C_{T,1}^\omega - EC_{T,1})(R^* M_1^\omega - 1)\right] - \eta\left[EC_{T,1} - C_{T,1}^\omega\right] \\
& - \mu^\omega \left[C_{T,1}^\omega - Y_{T,1}^\omega + (1+R^*)\bar{I} - R^* W_0 + N\left(C_{T,1}^\omega - (1-\rho)EC_{T,1}\right) - \frac{\bar{Y}_T - C_{T,1}^\omega}{R^*}\right]\Bigg\}
\end{aligned}$$

where μ^ω is the shadow value of relaxing the constraint on first period consumption in state ω, ν is the shadow value of the required risk compensation per unit N of insurance and η is the shadow value of increasing expected consumption $EC_{T,1}$ by one unit. This results in the following first-order conditions:

$$\begin{aligned}
\text{FOC}(C_{T,1}^\omega): \quad & u'(C_{T,1}^\omega) = \mu^\omega(1 + N + 1/R^*) + \nu(R^* M_1^\omega - 1) - \eta \quad \forall\omega \\
\text{FOC}(N): \quad & E\left\{\mu^\omega C_{T,1}^\omega\right\} = (1-\rho)E\left\{EC_{T,1}]\right\} \\
\text{FOC}(\rho): \quad & E[\mu^\omega] N EC_{T,1} = -\nu EC_{T,1} \\
\text{FOC}(EC_{T,1}): \quad & E[\mu^\omega] N(1-\rho) = \nu\rho + \eta
\end{aligned}$$

The first equation describes the consumption allocation in each state of the world. Agents' marginal utility of consumption has to equal $1 + N + 1/R^*$ times the shadow

price of relaxing the consumption constraint[12] plus the expected costs from increasing the risk premium, which depends on international investors' pricing kernel in that state, minus the costs from decreasing the average level of consumption. The second condition captures the optimal amount of insurance by equalizing the benefit and costs of the marginal unit of insurance. The last two first order conditions can be used to obtain the following expressions for the shadow prices

$$\nu = -N \cdot E[\mu^\omega] \quad \text{and} \quad \eta = N \cdot E[\mu^\omega] \tag{22}$$

We substitute these into the FOC($C^\omega_{T,1}$) to obtain

$$(1+\beta+N)\mu^\omega = u'(C^\omega_{T,1}) + NE\mu^\omega R^* M^\omega_1$$

Replacing this μ^ω in turn into the FOC(N) and dividing through the constant $(1+\beta+N)$ yields

$$E\left\{\left[u'(C^\omega_{T,1}) + NE\mu^\omega R^* M^\omega_1\right]\left[C^\omega_{T,1} - (1-\rho)EC_{T,1}\right]\right\} = 0$$

In order to simplify this equation, let us use the following lemma.

Lemma 1 *The expectation* $E\left\{bR^* M^\omega_1[C^\omega_{T,1} - (1-\rho)EC_{T,1}]\right\}$ *vanishes for any constant* b.

Proof. Opening up the square brackets and collecting terms, we find that

$$\begin{aligned} E\left\{R^* M^\omega_1[C^\omega_{T,1} - (1-\rho)EC_{T,1}]\right\} =& \rho R^* E[M^\omega_1]EC_{T,1} + R^*\mathrm{Cov}(M^\omega_1, C^\omega_{T,1}) = \\ =& \rho EC_{T,1} - \rho EC_{T,1} = 0 \end{aligned}$$

where the last step follows from condition (11) determining the equilibrium risk premium on local currency debt. Multiplying the expression by any finite constant leaves the conclusion unaffected. The economic reason for this result is that international investors are – by definition – indifferent between the random payoff $C^\omega_{T,1}$ and its certainty equivalent $(1-\rho)E[C^\omega_{T,1}]$. ■

Applying this lemma, we find that the social planner's optimal level of local currency debt is determined by the condition

$$\mathrm{Cov}(u'(C^\omega_{T,1}), C^\omega_{T,1}) = -\rho E[C^\omega_{T,1}]E[u'(C^\omega_{T,1})] \tag{23}$$

This expression is already familiar from the competitive equilibrium case. The solution of the constrained social planner is given by the 4 equations (11), (13), (20) and (23) in the 4 variables $(\rho, N, F^\omega_2, C^\omega_{T,1})$. These conditions are identical to the ones in the previous subsection determining the competitive equilibrium. We can summarize this result in the following proposition.

Proposition 2 *The equilibrium allocations of the constrained social planner and of agents in the decentralized equilibrium coincide.*

The constrained social planner thus cannot improve on the market outcome. Let us next analyze the case of an unconstrained social planner who can determine all of agents' decisions.

[12] This is because increases in $C_{T,1}$ entail higher repayments on the contingent asset and are smoothed over the ensuing periods. The exogenous components of the constraint on consumption (e.g. $Y^\omega_{T,1}$) has to rise by $1 + N + 1/R^*$ units for actual consumption to increase by one unit.

2.8 Social Planner

In the following analysis, the social planner coordinates not only the portfolio composition in the emerging market economy, but also agents' intertemporal pattern of consumption.

We will show that in general, the social optimum in this setup differs from the previous two cases. The reason is that, although the social planner faces the same assets to insure consumption, she recognizes that by altering agents' intertemporal consumption allocation, she can affect the relative price of non-tradable goods and therefore the relative payoff of the contingent asset. Deviating from agents' Euler equation in this way comes at a second-order cost, but provides a first-order insurance benefit to international investors, which they will pass on to borrowers in the form of a lower risk premium. In other words, the social planner mitigates the incompleteness of risk markets through a second-best intervention in the intertemporal allocation of consumption, which affects the relative payoff of the risky asset. In doing so, she can improve risk sharing between borrowers and investors. This is a general equilibrium effect that small agents do not internalize.[13]

Analytically, we drop the constraint on F_2^ω from the constrained social planner's maximization problem (21) and include the utility of consumption in both periods as the objective. The resulting Lagrangian is

$$\begin{aligned}\mathcal{L}^{SP} = E\Big\{ & u\left(C_{T,1}^\omega\right) + \beta u\left(\bar{Y}_T - R^* F_2^\omega\right) \\ & - \mu^\omega \left[C_{T,1}^\omega - Y_{T,1}^\omega + (1+R^*)\bar{I} - R^* W_0 + N\left(C_{T,1}^\omega - (1-\rho)EC_{T,1}\right) - F_2^\omega\right] \\ & - \nu\left[\rho EC_{T,1} + (C_{T,1}^\omega - EC_{T,1})(R^* M_1^\omega - 1)\right] - \eta\left[EC_{T,1} - C_{T,1}^\omega\right]\Big\} \end{aligned} \tag{24}$$

Compared to the previous case of the constrained social planner, the first order condition FOC($C_{T,1}^\omega$) changes, and the condition FOC(F_2^ω) is added, since the social planner has to decide on the optimal amount of second period borrowing:

$$\begin{aligned} &\text{FOC}(C_{T,1}^\omega): && u'(C_{T,1}^\omega) = \mu^\omega(1+N) + \nu(R^* M_1^\omega - 1) - \eta \quad \forall\omega \\ &\text{FOC}(F_2^\omega): && \beta R^* u'(C_{T,2}^\omega) = \mu^\omega \quad \forall\omega \end{aligned}$$

In the FOC($C_{T,1}^\omega$), the social planner internalizes the effect that making consumption more countercyclical to investors' pricing kernel M_1^ω reduces the risk premium because of its positive insurance effect on investors. The condition FOC(F_2^ω) implies that the social planner no longer equalizes $u'(C_{T,1}^\omega) = u'(C_{T,2}^\omega)$, but uses deviations from this rule as a second-best means of making up for missing insurance markets. The FOC(ρ) and FOC($EC_{T,1}$) are unchanged compared to the constrained social planner's case and imply that $\nu = -NE[\mu^\omega]$ and $\eta = NE[\mu^\omega]$, as given in (22). Substituting the results into the FOC($C_{T,1}^\omega$), we obtain that

$$\mu^\omega = \frac{u'(C_{T,1}^\omega) + NE[\mu^\omega]R^* M_1^\omega}{1+N} \tag{25}$$

[13]For a more general discussion of this externality see e.g. Stiglitz (1982).

Economically, μ^ω is the sum of the benefit in terms of additional consumption from relaxing the constraint on $C^\omega_{T,1}$ and the cost in terms of raising the risk premium. Using this μ^ω in the FOC(N), where the term with M^ω_1 vanishes because of lemma 1, and performing the same steps as above in the constrained social planner's problem, we obtain

$$\text{Cov}(u'(C^\omega_{T,1}), C^\omega_{T,1}) = -\rho E[C^\omega_{T,1}]E[u'(C^\omega_{T,1})] \tag{26}$$

In other words, the condition determining the optimal portfolio structure is the same as in both the competitive equilibrium and the constrained social planner's regime.

On the other hand, the social planner internalizes that the exchange rate is determined by agents' consumption allocations $C^\omega_{T,1}$, and that these in turn depend not only on the productivity shock and the portfolio structure, but also on her borrowing choices F^ω_2, which are determined by the first order condition $u'(C^\omega_{T,2}) = \mu^\omega$. She can raise investors' payoff in a given state ω by borrowing more F^ω_2, which raises agents' disposable income, their aggregate demand, and thus appreciates the real exchange rate. By the same token, reducing F^ω_2 depreciates the real exchange rate in that state.

If the emerging market economy and international investors are only exposed to a common risk factor, then risk markets between borrowers and investors are complete and the optimal amount of local currency debt L_1 can deliver the optimum insurance outcome. In that case, the solution to the decentralized equilibrium and the social planner's solution coincide. Let us define this situation formally as

Definition 1 *An emerging market's productivity shocks $Y^\omega_{T,1}$ and international investors' pricing kernel M^ω_1 are driven by a single common risk factor if fluctuations in the two variables are perfectly correlated, i.e. if*

$$|\text{Corr}(Y^\omega_{T,1}, M^\omega_1)| = 1 \tag{27}$$

Note that this condition is trivially satisfied if there are only two states of the world, i.e. if $\Omega = \{\omega_1, \omega_2\}$, and if $Y^\omega_{T,1}, M^\omega_1$ are non-degenerate. This is because in such a probability space, we can find two suitable scalars a and b for any two random variables X^ω, Y^ω such that $X^\omega = a + bY^\omega \ \forall\omega$.

On the other hand, there are two ways in which the condition can be violated. First, if some component of the emerging market economy's productivity shock is uncorrelated with investors' pricing kernel, investors would provide free insurance against this risk factor. Knowing this, the social planner would decrease her borrowing in states with an uncorrelated negative productivity shock. This would depreciate the exchange rate and thereby lower the contingent repayment mandated by agents' holdings of the contingent asset. The opposite considerations would apply during uncorrelated positive productivity shocks. Overall, this entails better insurance for domestic agents than the pure intertemporal consumption smoothing rule $u'(C^\omega_{T,1}) = \beta R^* u'(C^\omega_{T,2})$.

Secondly, if some component of investors' pricing kernel is uncorrelated with the productivity shock in the emerging market, then investors would be willing to pay borrowers to take on some of that risk. Borrowers would be willing to carry it against an appropriate compensation, which would take the form of a reduction in investors'

interest rate. The social planner thus borrows more F_2^ω in states in which international investors are affected by an uncorrelated negative shock; this appreciates the exchange rate in these states and increases the payoffs on the contingent asset to investors. In return for this insurance, investors demand a lower risk premium ρ.

Before summarizing these findings in a proposition, let us note the following lemma. (We have relegated the proof to the mathematical appendix.)

Lemma 2 *Let $X^\omega, Y^\omega, Z^\omega$ be scalar random variables that are perfectly correlated, such that $|\mathrm{Corr}(X^\omega, Y^\omega)| = |\mathrm{Corr}(Z^\omega, Y^\omega)| = 1$. Then $E[X^\omega] = E[Z^\omega]$ and $E[X^\omega Y^\omega] = E[Z^\omega Y^\omega]$ if and only if $X^\omega = Z^\omega$.*

Note that under the given assumptions, the condition $E[X^\omega Y^\omega] = E[Z^\omega Y^\omega]$ can also be written as $E[(X^\omega - Z^\omega)Y^\omega] = \mathrm{Cov}(X^\omega - Z^\omega, Y^\omega) = 0$.

Proposition 3 *If an emerging market's productivity shocks and investors' pricing kernel are driven by a single common risk factor and borrowers' utility function is quadratic, then the decentralized equilibrium coincides with the social planner's solution.*

Proof. $Y_{T,1}^\omega$ is the risk factor underlying the emerging market economy's productivity shocks. If there is only one risk factor driving both the emerging market economy and international investors' pricing kernel, then $|\mathrm{Corr}(Y_{T,1}^\omega, M_1^\omega)| = 1$. Given that borrowers' utility function is quadratic, all components of F_2^ω, $C_{T,1}^\omega$ and μ^ω are linear combinations of the two random variables and hence also perfectly correlated with $Y_{T,1}^\omega$. Note that we can re-write equation (25) as

$$\mu^\omega = u'(C_{T,1}^\omega) - \frac{N}{1+N} E[u'(C_{T,1}^\omega)] \left\{ \frac{u'(C_{T,1}^\omega)}{E[u'(C_{T,1}^\omega)]} - R^* M_1^\omega \right\}$$

If the term in curly brackets is zero, then the social planner's borrowing choices coincide with that of decentralized agents. We use lemma 2 to establish that this is true under the assumptions of the proposition. According to the lemma, it is sufficient to show that $E\left\{\frac{u'(C_{T,1}^\omega)}{E[u'(C_{T,1}^\omega)]}\right\} = E[R^* M_1^\omega]$ and $\mathrm{Cov}\left(\frac{u'(C_{T,1}^\omega)}{E[u'(C_{T,1}^\omega)]} - R^* M_1^\omega, C_{T,1}^\omega\right) = 0$ in order to prove that the term in square brackets vanishes and $\mu^\omega = u'(C_{T,1}^\omega)$. The first equality holds trivially. The second equality can be transformed to

$$\mathrm{Cov}\left(u'(C_{T,1}^\omega), C_{T,1}^\omega\right) = \mathrm{Cov}\left(C_{T,1}^\omega, R^* M_1^\omega\right) E[u'(C_{T,1}^\omega)]$$

The covariance term on the right hand side can be replaced by the risk premium according to equation (11), yielding the familiar condition

$$\mathrm{Cov}(u'(C_{T,1}^\omega), C_{T,1}^\omega) = -\rho E[C_{T,1}^\omega] E[u'(C_{T,1}^\omega)]$$

We have already demonstrated in equation (26) above that this condition holds in equilibrium. Therefore $\mu^\omega = u'(C_{T,1}^\omega)$, and all other equilibrium allocations coincide with the solutions to the decentralized equilibrium. ■

As discussed above, if productivity shocks and investors' pricing kernel are at least partially driven by different risk factors, then this result breaks down. We can then substitute the shadow price μ^ω from (25) into the FOC(F_2^ω) to obtain

$$u'(C_{T,2}^\omega) = u'(C_{T,1}^\omega) - \frac{N}{1+N} E[u'(C_{T,1}^\omega)] \cdot \left\{ \frac{u'(C_{T,1}^\omega)}{E[u'(C_{T,1}^\omega)]} - R^* M_1^\omega \right\}$$

If the marginal utilities of the two agents are affected by different risk factors, then in general $\frac{u'(C_{T,1}^\omega)}{E[u'(C_{T,1}^\omega)]} \neq R^* M_1^\omega$. In states in which domestic agents' marginal utility is relatively higher, the social planner borrows less than indicated by the intertemporal consumption smoothing condition $u'(C_{T,1}^\omega) = u'(C_{T,2}^\omega)$. As a result the real exchange rate depreciates relative to that case, and domestic agents' repayments on the contingent asset are lower. If investors' stochastic discount factor is relatively higher, the opposite results apply.

3 Borrowing Constraints and Financial Crises

In the previous section we emphasized the role of the exchange rate as a tool of macroeconomic adjustment: in the case of a negative productivity shock, aggregate demand is low and a real depreciation increases demand for local non-tradable goods so as to clear markets. We also noted that this created a pecuniary externality associated to decentralized agents' choice of portfolio structure: the less insurance in the form of local currency debt decentralized agents take on, the more of the adjustment has to be carried out by exchange rates, i.e. the more volatile the local currency becomes.

High exchange rate volatility can impose real costs on the economy, and this has proven to be of particular importance for emerging markets. As we noted in the beginning of this paper, both the theoretical and empirical literature maintain that exchange rate depreciations can have strongly contractionary effects (see e.g. Krugman, 1998, 1999; Levy Yeyati, 2006). While the economic literature has identified a number of contractionary effects of depreciations (see e.g. Caves et al., 2002), this paper focuses on balance sheet effects, which have played a key role in the financial crises of the past decade.[14]

Contractionary balance sheet effects work through the following mechanism: When borrowers have contracted dollar liabilities, exchange rate depreciations simultaneously inflate the local currency value of their dollar debts and reduce the international value of local currency denominated income and collateral. Both factors deteriorate borrowers' balance sheets, and this in turn magnifies a variety of problems of asymmetric information that exist between borrowers and investors, such as moral hazard and adverse selection, and can cause or exacerbate credit rationing (see Stiglitz and Weiss, 1981).

When borrowing constraints bind, uncontingent debt introduces an accelerator effect into the economy: Exchange rate depreciations deteriorate agents' balance sheets,

[14]For a detailed discussion of the importance of balance sheet effects versus other channels of contractionary depreciation see e.g. Frankel (2005). Note that the results of this paper do not depend on the exact channel through which depreciations have contractionary effects.

which tightens their borrowing constraints and reduces aggregate demand. Reduced aggregate demand in turn depreciates the exchange rate further. As described in Mendoza (2005), the two effects then lead to a contractionary cycle of falling exchange rates and tightening borrowing constraints, similar to the debt deflation process in Fisher (1933).

Note that the effects of borrowing constraints are asymmetric. While the described debt deflation process entails that exchange rate depreciations have strongly negative welfare effects, there are no corresponding positive effects to appreciations in good states when borrowing constraints are not binding – loose constraints are loosened further. In such situations, higher exchange rate volatility imposes clear real costs on the economy.

In the given setup, agents' period 0 borrowing is fixed by the requirement that $\bar{I} = F_1 + L_1 + W_0$. For simplicity we assume that this amount always satisfies the period 0 borrowing constraint. However, agents can be subject to binding borrowing constraints in period 1. We assume that the amount of debt they can take on in period 1 is subject to the constraint

$$F_2^\omega + L_2^\omega \leq \kappa(Y_{T,1}^\omega + p_{N,1}^\omega \bar{Y}_N) \tag{28}$$

This specification implies that agents' maximum amount of borrowing is linear in their income. Such borrowing constraints are common in the macroeconomic literature.[15] Lenders, for example, often put an important weight on current income to determine the maximum amount of loans they are willing to grant, especially in mortgage and consumer credit markets (Arellano and Mendoza, 2003). In a more general setup, we could also include the amounts of production and/or consumption next period as parameters in the borrowing constraint. However, the exact source of the credit market imperfection is not essential to the results in this paper, and the given linear specification offers the benefit of analytical simplicity.

As in the benchmark model, we assume that borrowers' optimal liability structure is such that their consumption is positive in all states and there is no bankruptcy. Furthermore, we limit financial contracts to one period contracts so as to guarantee that the borrowing constraints apply to the whole firm, not only to marginal borrowing.

The term $p_{N,1}^\omega$ in the collateral constraint is what can make exchange rate depreciations contractionary. A falling real exchange rate $p_{N,1}^\omega$ reduces the dollar value of non-tradable collateral and makes investors less willing to lend to domestic agents. If the constraint becomes binding, domestic agents' borrowing has to contract and their disposable income falls. As we noted earlier, this entails a further depreciation in $p_{N,1}^\omega$, which makes borrowing constraints even tighter, and so forth. In short, a Fisherian debt deflation process is set in motion. We impose the restriction on our parameters that $\kappa\varsigma < 1$ so as to guarantee that the feedback effects between exchange rate and

[15] See for example Mendoza (2005); Mendoza and Smith (2006). Aghion et al. (1999) derive a linear credit multiplier from a moral hazard problem among borrowers, who face an incentive to strategically default on their loans. Higher wealth implies that borrowers have more to lose from investors' efforts to collect, and they thus face a lower incentive to default. A related approach is presented in Bernanke et al. (1999), who show borrowing costs increase with falling net worth because of the higher associated probability of costly bankruptcy. As in our specification, this leads to a supply of debt that decreases in agents' income.

borrowing constraints become smaller from round to round and the economy converges to a non-degenerate equilibrium after a shock.

In the model presented here, the initial shock that triggers the described debt deflation process is a shock to tradable production $Y^{\omega}_{T,1}$, which lowers agents' income and therefore depreciates the exchange rate. In the spirit of the sudden stops described by Calvo (1998), borrowing constraints could also tighten because of a shock to investors' willingness to supply credit, as captured e.g. by the credit multiplier κ. Both kinds of shocks can cause borrowing constraints to suddenly bind, and can set in motion the described Fisherian debt deflation mechanism.

3.1 Competitive Equilibrium

Agents in the competitive equilibrium solve the same maximization problem as in the unconstrained case (8) of the previous section, augmented by the borrowing constraint defined in (28). The resulting Lagrangian is[16]

$$\mathcal{L}^{CE} = E\Big\{u(Y^{\omega}_{T,1} - (1+R^*)\bar{I} + R^*W_0 - R^*L_1[p^{\omega}_{N,1} - (1-\rho)E(p^{\omega}_{N,1})] + F^{\omega}_2) + \\ + \beta u(\bar{Y}_T - R^*F^{\omega}_2) - \lambda^{\omega}[F^{\omega}_2 - \kappa(Y^{\omega}_{T,1} + p^{\omega}_{N,1}\bar{Y}_N)]\Big\}$$

The corresponding first order conditions are

$$\text{FOC}(F^{\omega}_2) : u'(C^{\omega}_{T,1}) = \beta R^* u'(C^{\omega}_{T,2}) + \lambda^{\omega}$$
$$\text{FOC}(L_1) : E\left\{u'(C^{\omega}_{T,1})[p^{\omega}_{N,1} - (1-\rho)E(p^{\omega}_{N,1})]\right\} = 0$$

We note first that, despite of the additional constraint, the FOC(L_1) can be transformed to yield the same solution as condition (18) in the previous section

$$\text{Cov}(u'(C^{\omega}_{T,1}), C^{\omega}_{T,1}) = \rho E[C^{\omega}_{T,1}]E[u'(C^{\omega}_{T,1})] \tag{29}$$

As long as the collateral constraint is not binding, agents' choice of second period borrowing equalizes the marginal utilities of period 1 and period 2 consumption. On the other hand, if the borrowing constraint is binding, agents borrow the maximum amount possible. In short, this implies that in equilibrium

$$F^{\omega}_2 = \min\left\{\frac{\bar{Y}_T - C^{\omega}_{T,1}}{R^*}, \kappa\left(Y^{\omega}_{T,1} + \varsigma C^{\omega}_{T,1}\right)\right\} \tag{30}$$

Applying the transformation $N = L_1 \frac{R^*}{(1-\rho)E[C^{\omega}_{T,1}]}$, we can use these F^{ω}_2 to express agents' optimal first period consumption $C^{\text{con}}_{T,1}$ in constrained states and $C^{\text{unc}}_{T,1}$ in unconstrained states of the world as

$$C^{\text{con}}_{T,1} = \frac{(1+\kappa)Y^{\omega}_{T,1} - (1+R^*)\bar{I} + R^*W_0 + (1-\rho)NE[C^{\omega}_{T,1}]}{1+N-\kappa\varsigma} \tag{31}$$

$$\text{and } C^{\text{unc}}_{T,1} = \frac{\bar{Y}_T + R^*Y^{\omega}_{T,1} - R^*(1+R^*)\bar{I} + (R^*)^2W_0 + (1-\rho)NR^*E[C^{\omega}_{T,1}]}{1+R^*(1+N)} \tag{32}$$

[16] We have made the usual simplifications, i.e. we set $C^{\omega}_{N,1} = \bar{Y}_N$, substituted the utility function from (12) and assumed w.l.o.g. that $L^{\omega}_2 = 0$.

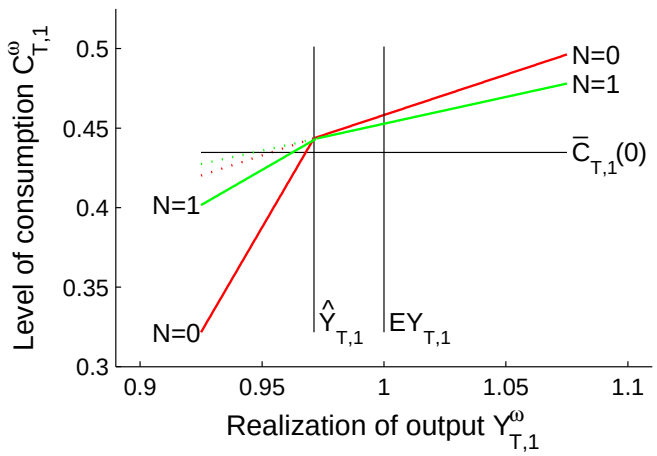

Figure 4: The figure depicts the level of consumption $C^{\omega}_{T,1}$ as a function of the output shock $Y^{\omega}_{T,1}$ for several different levels of insurance N. As can be seen, more insurance makes consumption less sensitive to the shock, and particularly so in those states where the credit constraint is binding. As a result, the variance of consumption is reduced.

A direct implication is the following result:

Proposition 4 *Under binding borrowing constraints, consumption is more sensitive to shocks. Borrowing in the contingent asset can reduce the sensitivity of consumption to shocks, and it can do so more strongly when borrowing constraints are binding. As a result, borrowing in the contingent asset rather than uncontingent debt reduces the volatility of consumption.*

Proof. The sensitivity of consumption to shocks can be calculated as the derivative of the respective consumption variable to $Y^{\omega}_{T,1}$.

$$\frac{dC^{\text{con}}_{T,1}}{dY^{\omega}_{T,1}} = \frac{1+\kappa}{1+N-\kappa\varsigma} \qquad \frac{d^2C^{\text{con}}_{T,1}}{dY^{\omega}_{T,1}dN} = -\frac{1+\kappa}{(1+N-\kappa\varsigma)^2}$$

$$\frac{dC^{\text{unc}}_{T,1}}{dY^{\omega}_{T,1}} = \frac{R^*}{1+R^*(1+N)} \qquad \frac{d^2C^{\text{unc}}_{T,1}}{dY^{\omega}_{T,1}dN} = -\left(\frac{R^*}{1+R^*(1+N)}\right)^2$$

Clearly, constrained consumption is more sensitive to output shocks. This is because when borrowing constraints are binding, an increase in income is fully consumed and, in addition, increases agents' collateral and therefore the amount that they can borrow. When borrowing constraints are loose, on the other hand, only a fraction of the increase in income is consumed; the rest of it is saved so as to smooth consumption over the ensuing period.

Insurance mitigates the impact of shocks on consumption, and since this impact is stronger in borrowing constrained states, the contingent asset has a stronger mitigating impact in those states. ■

Figure 4 illustrates the effects of contingent debt on consumption volatility graphically. In unconstrained states, insurance mitigates the sensitivity of consumption to

shocks to roughly the same extent as in the benchmark case without credit constraints in figure 1 on page 11. On the other hand, in those states where borrowing constraints are binding, i.e. to the left of the vertical $\hat{Y}_{T,1}$-line in the figure, the response of consumption to output shocks is magnified because of the debt deflation effect. The contingent asset mitigates these feedback effects and therefore smoothes the response.

Let us indicate the threshold values of output and consumption at which the borrowing constraint is marginally binding by hats, i.e. by $\hat{Y}_{T,1}$ and $\hat{C}_{T,1}$. For realizations of the shock with $Y^{\omega}_{T,1} < \hat{Y}_{T,1}$, the constraint is binding, for realizations with $Y^{\omega}_{T,1} \geq \hat{Y}_{T,1}$ it is loose. At the cutoff value $\hat{Y}_{T,1}$, the constrained and the unconstrained levels of consumption $C^{\text{con}}_{T,1}$ and $C^{\text{unc}}_{T,1}$ are equal and the constraint is marginally binding. We find $\hat{Y}_{T,1}$ as the value of $Y^{\omega}_{T,1}$ for which the two expressions (31) and (32) are equal.

$$\hat{Y}_{T,1} = \frac{(1+\kappa\varsigma R^*)\left[(1+R^*)\bar{I} - R^* W_0 - (1-\rho)NE[C^{\omega}_{T,1}]\right] + \bar{Y}_T\left(1+N-\kappa\varsigma\right)}{1+\kappa\left[1+R^*\left(1+N-\varsigma\right)\right]}$$

Note that $F(\hat{Y}_{T,1})$ represents the probability that borrowers will be constrained. Taking the derivative of $\hat{Y}_{T,1}$ with respect to N (details are provided in appendix D), we find that

$$\frac{d\hat{Y}_{T,1}}{dN} = \frac{1+\kappa\varsigma R^*}{1+\kappa\left[1+R^*\left(1+N+\varsigma\right)\right]} \cdot \left[\hat{C}_{T,1} - \bar{C}_{T,1}\right]$$

where $\bar{C}_{T,1} = \frac{d(1-\rho)NEC_{T,1}}{dN}$ is again the constant payoff that agents receive from increasing the amount of insurance N and forgoing the random realization of consumption $C^{\omega}_{T,1}$. The sign of the derivative $d\hat{Y}_{T,1}/dN$ depends on the sign of $\hat{C}_{T,1} - \bar{C}_{T,1}$, i.e. more insurance reduces the threshold if the net payoff of insurance in the marginally constrained state is positive. There are two factors that could potentially turn the sign of this derivative negative: First, if investors are strongly risk-averse, then the risk premium would be high and the certainty equivalent of consumption $(1-\rho)E[C^{\omega}_{T,1}]$ could fall below the threshold. Secondly, if the incidence of constraints is high, then the marginally constrained state $\hat{C}_{T,1}$ can be pushed above the certainty equivalent.

3.2 Social Planner

Individual agents regard the exchange rate, and therefore the dollar value of their collateral and the tightness of borrowing constraints, as exogenously given. The social planner, by contrast, internalizes that exchange rates are driven by aggregate macroeconomic outcomes and that agents' behavior influences the tightness of borrowing constraints. In particular, the social planner recognizes that increased insurance holdings make the exchange rate less volatile and thereby mitigates the potential costs of borrowing constraint. Analytically, we add the borrowing constraint (28) to the social planner's optimization problem (24) from the benchmark case of the previous section.

The corresponding Lagrangian is

$$\mathcal{L}^{SP} = E\Big\{u\left(C_{T,1}^{\omega}\right) + \beta u\left(\bar{Y}_T - R^* F_2^{\omega}\right) - \lambda^{\omega}\left[F_2^{\omega} - \kappa\left(Y_{T,1}^{\omega} + \varsigma C_{T,1}^{\omega}\right)\right]$$
$$- \mu^{\omega}\left[C_{T,1}^{\omega} - Y_{T,1}^{\omega} + (1+R^*)\bar{I} - R^* W_0 + N\left(C_{T,1}^{\omega} - (1-\rho)EC_{T,1}\right) - F_2^{\omega}\right]$$
$$- \nu\left[\rho EC_{T,1} + (C_{T,1}^{\omega} - EC_{T,1})(R^* M_1^{\omega} - 1)\right] - \eta\left[EC_{T,1} - C_{T,1}^{\omega}\right]\Big\}$$

Here λ^{ω} is the multiplier on the borrowing constraint. The resulting first-order conditions on $C_{T,1}^{\omega}$ and F_2^{ω} are:

$$\text{FOC}(C_{T,1}^{\omega}): \qquad u'(C_{T,1}^{\omega}) + \lambda^{\omega}\kappa\varsigma = \mu^{\omega}(1+N) + \nu(R^* M_1^{\omega} - 1) - \eta \quad \forall\omega$$
$$\text{FOC}(F_2^{\omega}): \qquad \beta R^* u'(C_{T,2}^{\omega}) + \lambda^{\omega} = \mu^{\omega} \quad \forall\omega$$

The first-order conditions FOC(N), FOC(ρ) and FOC($EC_{T,1}$) are unchanged from the benchmark case without borrowing constraints, and the latter two equations give rise to the same two expressions $\nu = -NE[\mu^{\omega}]$ and $\eta = NE[\mu^{\omega}]$ as in equation (22) of that section. Substituting these as well as the FOC(F_2^{ω}) into the FOC($C_{T,1}^{\omega}$) yields

$$(1+N)\mu^{\omega} = u'(C_{T,1}^{\omega}) + \lambda^{\omega}\kappa\varsigma + NE[\mu^{\omega}]R^* M_1^{\omega}$$

In conjunction with the first order condition FOC(N), where the term with M_1^{ω} vanishes according to lemma 1, we then obtain

$$\text{Cov}(u'(C_{T,1}^{\omega}), C_{T,1}^{\omega}) = -\rho E[C_{T,1}^{\omega}]E[u'(C_{T,1}^{\omega})] - \kappa\varsigma E\left[\lambda^{\omega}\left(C_{T,1}^{\omega} - (1-\rho)E[C_{T,1}^{\omega}]\right)\right]$$

The first term on the right-hand side of this expression is identical to the equilibrium condition (29) for insurance in the decentralized equilibrium. The second term captures the effects of insurance on the incidence of collateral constraints, which decentralized agents do not internalize. Let us define the term θ to capture this externality and re-write the condition for the social planner's optimum amount of insurance as

$$\text{Cov}(u'(C_{T,1}^{\omega}), C_{T,1}^{\omega}) = -(\rho - \theta)E[C_{T,1}^{\omega}]E[u'(C_{T,1}^{\omega})] \tag{33}$$
$$\text{where} \quad \theta = -\kappa\varsigma \cdot \frac{E\left[\lambda^{\omega}\left(C_{T,1}^{\omega} - (1-\rho)E[C_{T,1}^{\omega}]\right)\right]}{E[C_{T,1}^{\omega}]E[u'(C_{T,1}^{\omega})]} \tag{34}$$

λ^{ω} is the shadow value of relaxing the borrowing constraint in period ω, i.e. the wedge that the constraint introduces into the agent's Euler equation. It is positive in low productivity states $Y_{T,1}^{\omega} < \hat{Y}_{T,1}$ in which borrowing constraints are binding, and zero otherwise. When constraints bind in low productivity states, consumption is generally also small, so that $C_{T,1}^{\omega} < (1-\rho)E[C_{T,1}^{\omega}]$, as long as investors' risk premium is not too large. The product inside of the expectations operator in the expression for θ is thus negative in constrained states and zero in unconstrained states, in which the shadow value on the borrowing constraint is $\lambda^{\omega} = 0$. This implies that the externality term θ is generally positive.

Using the quadratic utility approximation from earlier[17], we can describe the social planner's choice of insurance by

$$\mathrm{Var}(C^{\omega}_{T,1}) = (\rho - \theta)E[u'(C^{\omega}_{T,1})]E[C^{\omega}_{T,1}]$$

For a given risk premium ρ and a positive externality term θ, the social planner's solution entails less volatility in consumption than the decentralized equilibrium. The social planner reaches this outcome by contracting more insurance than decentralized agents. In equilibrium, this leads to a reduction in consumption volatility, and by extension to a decrease in exchange rate volatility and a reduction in international investors' risk premium.

As in our discussion of the sensitivity of $\hat{Y}_{T,1}$ to the amount of insurance N, there are exceptional circumstances in which the opposite results hold: when international investors are strongly risk-averse and the probability of binding borrowing constraints is very high, the certainty equivalent $(1-\rho)E[C^{\omega}_{T,1}]$ can be less than consumption in those constrained states in which insurance is supposed to alleviate agents' borrowing constraints. In the extreme case that $(1-\rho)E[C^{\omega}_{T,1}] < \min\left\{C^{\omega}_{T,1}\right\}$, more insurance would reduce consumption in all states – in such a situation, domestic agents unambiguously benefit from reducing their holdings of insurance. They would correspondingly increase their uncontingent debts and sell insurance to international investors, who would be willing to pay them a substantial premium for taking on this risk. However, decentralized agents would not internalize that the higher consumption obtained from these earnings would also mitigate collateral constraints through their effect on exchange rates; therefore they might hold too little uncontingent debt.

More generally, we can distinguish between two effects of insurance: the insurance effect consists of the contingent payment $-(C^{\omega}_{T,1} - E[C^{\omega}_{T,1}])$; it always reduces the overall costs of borrowing constraints by smoothing income, i.e. by providing positive payoffs in low states. The (negative) income effect of insurance is captured by the uncontingent payment of the risk premium $\rho E[C^{\omega}_{T,1}]$; it lowers consumption in all states and increases the incidence and tightness of binding constraints.

Decentralized agents only trade off the benefits and costs of insurance against consumption risk in choosing their portfolio structure. The social planner, on the other hand, also considers the insurance and income effects on the incidence of collateral constraints. Since the impact of these constraints is asymmetric (loosening binding constraints in low states is beneficial, while tightening loose constraints in high states is costless), the social planner's optimum entails more insurance, than the decentralized equilibrium under regular circumstances. The following is the technical condition to guarantee that the externality term θ is positive:

Condition 1 *The externality term is positive $\theta > 0$ if and only if*

$$\int_0^{(1-\rho)E[C^{\omega}_{T,1}]} \lambda^{\omega}\left[(1-\rho)E(C^{\omega}_{T,1}) - C^{\omega}_{T,1}\right] dC^{\omega}_{T,1} > \int_{(1-\rho)E[C^{\omega}_{T,1}]}^{\infty} \lambda^{\omega}\left[C^{\omega}_{T,1} - (1-\rho)E(C^{\omega}_{T,1})\right] dC^{\omega}_{T,1}$$

[17]The general result holds for all neoclassical utility functions, i.e. the relationship between the covariance term in (33) and the variance of consumption is always monotonic, but not necessarily linear.

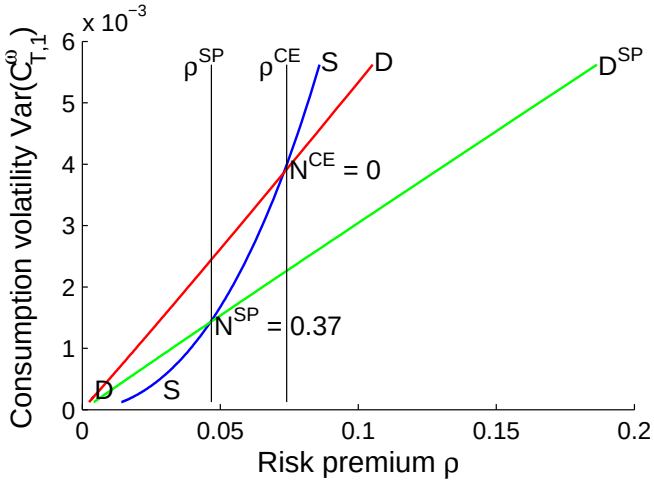

Figure 5: The figure depicts investors' and private borrowers' optimality loci SS and DD, as well as the social planner's optimality locus DD^{SP}. The competitive equilibrium and the social planner's solution are determined by the intersection of international investors' locus with domestic agents' or the social planner's optimality locus, here marked by the vertical lines ρ^{SP} and ρ^{CE}. The resulting equilibrium values of N are indicated next to the intersection points.

As long as the left-hand side, i.e. the sum of all positive insurance payoffs, weighted by the shadow prices of relaxing the constraint in the respective state of the world, exceed the sum of weighted negative payoffs, more insurance reduces borrowing constraints and θ is therefore positive.

Example 1 If $\hat{C}_{T,1} < (1-\rho)E[C^{\omega}_{T,1}]$, then the payoffs to insurance in constrained states are always positive. In the inequality in condition 1, the left hand-side is positive and the right-hand side is zero. Hence the decentralized equilibrium in such an economy is always characterized by excessive holdings of uncontingent debt.

Example 2 In an economy with only two states, any allocation in the decentralized equilibrium has to satisfy $C^{L}_{T,1} < (1-\rho)E[C^{\omega}_{T,1}]$. Otherwise, lowering the amount of insurance would increase consumption in both states of the world because of a positive income effect, and make agents unambiguously better off. If agents are only constrained in the low state, then increasing the amount of insurance would unambiguously alleviate these borrowing constraints without impacting borrowing in the high state. Consequently, condition 1 is always satisfied in such an economy and decentralized agents would borrow excessively in dollars.

The implications for the optimum amount of local currency debt are the following:

Proposition 5 *If the externality term θ is positive, then the decentralized equilibrium is characterized by too little insurance and excessive holdings of uncontingent debt.*

Graphically, a positive externality term θ implies that the social planner's optimality locus is to the right of decentralized agents' optimality locus in figure 5. As can be seen, the social planner's equilibrium then exhibits lower consumption volatility and a lower risk premium.

4 Policy Implications

The externality in this paper arises because of the interaction of uncontingent debts with borrowing constraints. Hence the first-best policy solution would be to address the capital market imperfections that underlie these constraints, for example through better enforcement of creditor rights, streamlined bankruptcy proceedings etc. However, in the short to medium term, policymakers have to take many of the features of the institutional framework of emerging market economies as given.

Recognizing this constraint, a common policy prescription (see e.g. Goldstein and Turner, 2004) is that firms should match the currency denomination of their liabilities with that of their revenues. However, even if this rule is followed, the externality in my paper is still present: if e.g. firms in the tradable sector borrow in dollars, the volatility of the real exchange rate will be higher than if they borrowed in the form of contingent debt, and this will negatively impact the dollar value of everybody's local currency-denominated collateral in low states. Furthermore, some of the dollar denomination in firms' revenues might be fictitious, e.g. if the collateral that they receive from domestic clients in case of bankruptcy is non-tradable, such as real estate (Tirole, 2003).

Another key factor that gives rise to the externality is aversion to emerging market risk on the part of international investors. Without a risk premium, domestic borrowers would have no incentive to resort to uncontingent debt and cause externalities. In general, stronger institutions within emerging markets help to reduce international investors' risk aversion, but as discussed above this might not be achievable in the short run. A number of researchers have put forward proposals to deepen international markets in emerging market currencies by combining different currencies into one security with superior risk/return characteristics (see Dodd and Spiegel, 2005; Eichengreen and Hausmann, 2005). If this widens the pool of investors in emerging market currencies and therefore enhances risk diversification in international capital markets, the risk premium charged on individual domestic currencies could be reduced.

A policy prescription that has been widely followed during emerging market financial crises is to attempt to stabilize exchange rates (Calvo and Reinhart, 2002). The model in this paper discusses real exchange rates and assumes that the exchange rate is tied to the price of non-tradables, i.e. implicitly that monetary authorities stabilize the money price of non-tradable goods. In a more general setup, let us assume that the central bank stabilizes an index of tradable and non-tradable goods[18]

$$P = p_T^{\phi} p_N^{1-\phi}$$

where ϕ is the relative weight on tradables. In this formulation, the analysis of sections 2 and 3 of this paper captures the case $\phi = 0$.

The other polar case that $\phi = 1$ describes a central bank that stabilizes the money value of tradable goods, which, given a constant world market price of tradable goods, can be interpreted as a policy of pegging the exchange rate. Note that if a central bank credibly follows this policy, the contingent and uncontingent assets in our model are equivalent. As a consequence, the risk premium would disappear. At the same time, insurance could no longer be obtained: the economy forgoes the exchange rate as a

[18]For a detailed discussion of the mechanism involved, see e.g. Woodford (2003).

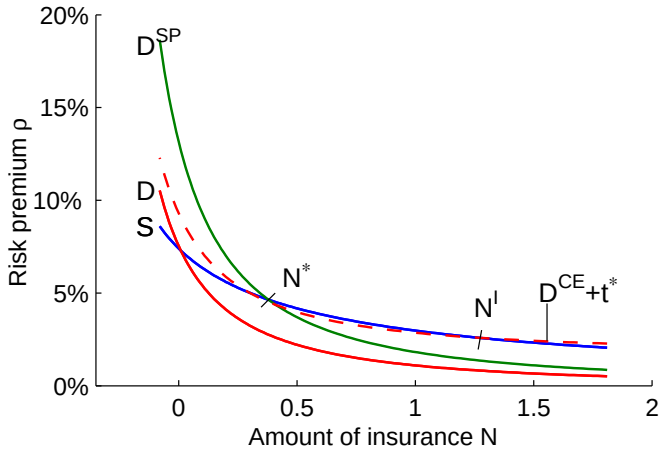

Figure 6: A linear tax t on uncontingent debt shifts the demand curve for insurance upwards in parallel, which can implement the social optimum N^*, but can also lead to the inefficient equilibrium N^I.

tool of macroeconomic adjustment. Still, the real exchange rate would fluctuate in this model, implying inflation in the money price of non-tradable goods in high productivity states and deflation in low productivity states. This situation corresponds to imposing the constraint $L_1 = 0$ in our benchmark model and would clearly reduce welfare in the economy.

On the other hand, if the peg is not perfectly credible, an interest rate differential between local currency and dollar will capture depreciation expectations and investors' risk aversion. Our externality result still holds in this case: domestic borrowers will want to save on investors' risk premium and engage in dollar borrowing, not recognizing that this will adversely affect the dollar value of their collateral in low productivity states, when central banks are most likely to abandon the peg.

More generally, the monetary authorities could stabilize any price index with $\phi \in [0, 1]$. The case $\phi = \sigma$, for example, corresponds to a policy of targeting constant consumer prices. As long as $\phi < 1$, exchange rates are counter-cyclical, contingent assets insures borrowers against aggregate productivity shocks, and the externality is still present. However, for $\phi \neq 0$ the exchange rate is a non-linear function of the shock and risk markets are no longer complete.

4.1 Optimal Tax on International Capital Flows

We showed above in section 3.2 that the socially optimal level of uncontingent liabilities is in general less than the amount that decentralized agents contract. It follows naturally that the social planner could make agents internalize the externality associated with their level of uncontingent debt by imposing a tax on it.

The optimal fixed tax t^* on uncontingent debt is such that in equilibrium, borrowers pay a spread of $\rho - \theta$ instead of ρ on insurance, so as to internalize the externality. For a tax t^* per unit of foreign currency, this implies that we require

$$[1 - (\rho - \theta)]E[R^{\omega}_{L,1}] = R^* + t^* \quad \text{or} \quad t^* = \theta E[R^{\omega}_{L,1}] \tag{35}$$

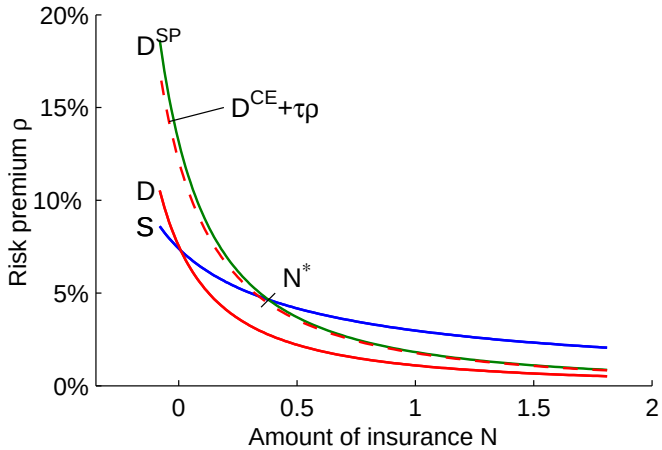

Figure 7: A tax τ that is proportional to the risk premium on the contingent asset implements the social optimum. It shift the demand curve for insurance upwards towards the social planner's demand and raises the equilibrium amount of insurance.

where the values of θ and $E[R^{\omega}_{L,1}]$ are the ones in the social planner's equilibrium. An example for such an optimal tax is shown in figure 6, for which we used the parameter values discussed in footnote 10 on page 11 and calibrated investors' risk aversion such that they demand a risk premium of 3% when $N = 1$. Borrowers' risk aversion is calibrated to yield a zero level of contingent debt in the competitive equilibrium. The optimal tax rate per unit of uncontingent assets in this example turns out to be $t^* = 1.75\%$.

Note that a constant tax t^* has the disadvantage that it leads to a situation of multiple equilibria, including an inefficient equilibrium with excessive insurance (in the figure denoted as N^I). In that equilibrium, the volatility of the economy is low because of the high amount of insurance; the tax t on uncontingent debt is much higher than the externality and induces borrowers to take on an inefficiely large amount of insurance. However, this equilibrium is unstable. The reason for its existence is that the size of the externality θ (in the figure the vertical difference between decentralized agents' and the social planner's demand curves) decreases approximately proportionally to the spread ρ, whereas the tax rate t^* is fixed.

This suggests that it would be more efficient to levy a tax that is proportional to the risk premium, i.e. a tax in the amount of $\tau\rho$. The optimal τ for such a tax could be determined using the condition

$$[1-(\rho-\theta)]E[R^{\omega}_{L,1}] = R^* + \tau\rho \quad \text{or} \quad \tau = \frac{\theta}{\rho}E[R^{\omega}_{L,1}] = \frac{\theta}{\rho(1-\rho)}R^* \tag{36}$$

where the values of θ and ρ are again the ones at the social planner's equilibrium. In the given example, the optimal $\tau = 38\%$. When we depict the shift in decentralized agents' demand that results from such a tax in a diagram, it almost coincides with the social planner's demand in figure 7.

In terms of implementation, there is a number of policy measures that have tax-like effects on uncontingent debt. These include the following:

- *Reserve requirements:* A country can impose reserve requirements on undesired forms of capital flows. In the simplest version, it would require investors to hold an amount of $\frac{t^*}{R^*-1}$ per unit lent in an account without remuneration, so that the lost amount of interest would equal the optimal tax rate. A similar approach was used by Chile to regulate short-term capital flows (see e.g. Gallego et al., 2002).

  A more refined version of this policy instrument could require international investors to hold an amount of τ per unit lent in an unremunerated account denominated in a contingent asset such as local currency (which could be inflation-indexed). In that case, the lost amount of interest would approximately equal the relative tax rate τ. Furthermore, investors would be obliged to carry some insurance even though their loan is uncontingent.

- *Tax Deductability:* In most tax legislations around the world, interest rate payments are tax deductible for corporations. Selectively limiting this deductability on dollar, but not domestic currency debt would be equivalent to an increase in the effective interest rate for foreign currency borrowers. A full elimination of the tax deductability of interest payments on dollar debt, for example, would be equivalent to a rise in the dollar interest rate by $\tau^C(R^* - 1)$, where τ^C represents the corporate tax rate applicable to that borrower.

- *Bankruptcy regulations:* Uncontingent debt can be penalized through differential treatment in bankruptcy regulations. Although we have not modeled the effects of bankruptcy in our framework, the model could easily be extended to allow for an unfavorable treatment of uncontingent debts in the case of bankruptcy, which would equal the tax t^* in expected value. A big benefit of such a setup might be that the probability of bankruptcy of borrowers in an economy varies over time, and that these variations would generally go in parallel with the incidenc of borrowing constraints and thus with the externality discussed here.

Most of these policy instruments could be circumvented via derivatives markets. Naturally, any regulations in debt markets would thus have to be accompanied by corresponding regulations in derivatives markets.

5 Conclusions

The goal of this paper was to analyze the portfolio decisions of decentralized agents in emerging markets and to demonstrate that they borrow excessively in the form of uncontingent assets because they fail to internalize the effects of their decision on the value of their collateral. The insurance effect of contingent liabilities reduces the volatility of agents' disposable income. In aggregate, this makes demand, the current account, and by extension exchange rates, less volatile.

Our main result was that this pecuniary externality introduces a distortion into the economy when individual agents are subject to collateral-dependent borrowing constraints: stable exchange rates mitigate the negative effects of a low productivity shock on the dollar value of each agent's collateral, making borrowing constraints less binding

and financial crises less likely. Decentralized agents fail to internalize this positive externality to insurance (or, equivalently, negative externality to uncontingent debt) and engage in too much uncontingent borrowing from a social point of view.

Excessive uncontingent debts make emerging market economies more volatile and increase both the incidence and the severity of financial crises. We have discussed a number of potential policy remedies to correct the distortion: While a linear tax on unontingent debt can result in multiple equilibria, we showed that a tax that is proportional to the risk premium, implementable for example through unremunerated reserve requirements, provided a better solution. Similar effects can be obtained through banking and bankruptcy regulations, or changes in the tax deductability of interest payments on uncontingent debt.

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A Proof of Lemma 1

Proof of Lemma 1. The proof from left to right works as follows. The assumptions on the correlations imply that we can write $X^\omega = a + bY^\omega$ and $Z^\omega = c + dY^\omega$ for some scalars a, b, c, d. Then the two equations in expectations read as

$$\begin{aligned} a + bE[Y^\omega] = c + dE[Y^\omega] \quad &\text{and} \quad aE[Y^\omega] + bE[(Y^\omega)^2] = cE[Y^\omega] + dE[(Y^\omega)^2] \\ \text{or} \quad (a-c) = (d-b)E[Y^\omega] \quad &\text{and} \quad (a-c)E[Y^\omega] = (d-b)E[(Y^\omega)^2] \end{aligned}$$

Eliminating alternatively $(a-c)$ and $(d-b)$ from the two equations yields

$$(a-c) = (a-c)\frac{(E[Y^\omega])^2}{E[(Y^\omega)^2]} \quad \text{and} \quad (d-b)(E[Y^\omega])^2 = (d-b)E[(Y^\omega)^2]$$

Note that, since Y^ω has non-zero correlation with other variables, its variance must be positive, and therefore $E[(Y^\omega)^2] > (E[Y^\omega])^2 > 0$. The equations above can then only be satisfied if $a = c$ and $b = d$, which implies that $X^\omega = Z^\omega$. The proof in the opposite direction is trivial. ■

B Derivation of $\bar{C}_{T,1}$ in Benchmark Model

We defined earlier that $\bar{C}_{T,1} = \frac{d(1-\rho)NE[C^\omega_{T,1}]}{dN}$ and re-wrote this as

$$\bar{C}_{T,1} = \frac{d(1-\rho)NE[C^\omega_{T,1}]}{dN} = (1-\rho)E[C^\omega_{T,1}] - NE[C^\omega_{T,1}]\frac{d\rho}{dN} + (1-\rho)N\frac{dE[C^\omega_{T,1}]}{dN}$$

$\bar{C}_{T,1}$ can be regarded as something like lenders' certainty equivalent of consumption, taking into account general equilibrium effects. (More precisely speaking, $\bar{C}_{T,1}$ is the increase in borrowers' fixed payoff $(1-\rho)NE[C^\omega_{T,1}]$ from contracting more local currency debt, taking into account all general equilibrium effects.

$\bar{C}_{T,1}$ consists of three different terms: The first one, $(1-\rho)E[C^\omega_{T,1}]$, captures that borrowers receive the expected value discounted by a risk premium in return for forgoing the random variable $C^\omega_{T,1}$. The second term, $NE[C^\omega_{T,1}]\frac{d\rho}{dN}$, internalizes that the risk premium actually falls the more insurance agents take on. The third term takes into account that, since agents have to pay for insuring, expected consumption itself falls the more insurance is taken on.

Let us analyze the two derivatives on the right hand side of this expression. We note first that we can write the two equations (11) and (16) defining the equilibrium values of ρ and $E[C^{\omega}_{T,1}]$ in vector form as

$$\begin{pmatrix} \rho \\ E[C^{\omega}_{T,1}] \end{pmatrix} = \begin{pmatrix} -\frac{(R^*)^2 \mathrm{Cov}\left(Y^{\omega}_{T,1}, M^{\omega}_1\right)}{E[C^{\omega}_{T,1}][1+R^*(1+N)]} \\ \frac{(1+R^*)\overline{Y}_{T,1}-(1+R^*)\bar{I}+R^*W_0}{1+R^*(1+\rho N)} \end{pmatrix}$$

Differentiating this system of equations with respect to N yields

$$\begin{pmatrix} 1 & \frac{\rho}{E[C^{\omega}_{T,1}]} \\ \frac{R^*NE[C^{\omega}_{T,1}]}{1+R^*(1+\rho N)} & 1 \end{pmatrix} \begin{pmatrix} \frac{d\rho}{dN} \\ \frac{dE[C^{\omega}_{T,1}]}{dN} \end{pmatrix} = - \begin{pmatrix} \frac{\rho R^*}{1+R^*(1+N)} \\ \frac{\rho R^* E[C^{\omega}_{T,1}]}{1+R^*(1+\rho N)} \end{pmatrix} \tag{37}$$

The determinant of the matrix that pre-multiplies the vector of differentials is

$$\left| \begin{pmatrix} 1 & \frac{\rho}{E[C^{\omega}_{T,1}]} \\ \frac{R^*NE[C^{\omega}_{T,1}]}{1+R^*(1+\rho N)} & 1 \end{pmatrix} \right| = 1 - \frac{\rho R^* N}{1+R^*(1+\rho N)} = \frac{1+R^*}{1+R^*(1+\rho N)}$$

We use this determinant to invert the matrix and left-multiply equation (37) by this inverse to obtain

$$\begin{aligned} \begin{pmatrix} \frac{d\rho}{dN} \\ \frac{dE[C^{\omega}_{T,1}]}{dN} \end{pmatrix} &= -\frac{1+R^*(1+\rho N)}{1+R^*} \begin{pmatrix} 1 & -\frac{\rho}{E[C^{\omega}_{T,1}]} \\ -\frac{R^*NE[C^{\omega}_{T,1}]}{1+R^*(1+\rho N)} & 1 \end{pmatrix} \begin{pmatrix} \frac{\rho R^*}{1+R^*(1+N)} \\ \frac{\rho R^* E[C^{\omega}_{T,1}]}{1+R^*(1+\rho N)} \end{pmatrix} = \\ &= -\frac{1+R^*(1+\rho N)}{1+R^*} \begin{pmatrix} \frac{\rho R^*}{1+R^*(1+N)} - \frac{\rho^2 R^*}{1+R^*(1+\rho N)} \\ -\frac{\rho (R^*)^2 NE[C^{\omega}_{T,1}]}{[1+R^*(1+\rho N)][1+R^*(1+N)]} + \frac{\rho R^* E[C^{\omega}_{T,1}]}{1+R^*(1+\rho N)} \end{pmatrix} = \\ &= -\begin{pmatrix} \rho R^* \frac{1+R^*(1+\rho N)-\rho-\rho R^*(1+N)}{[1+R^*(1+N)](1+R^*)} \\ \rho R^* E[C^{\omega}_{T,1}] \frac{1+R^*(1+N)-R^*N}{[1+R^*(1+N)](1+R^*)} \end{pmatrix} = \begin{pmatrix} -\frac{\rho(1-\rho)R^*}{1+R^*(1+N)} \\ \frac{\rho R^* E[C^{\omega}_{T,1}]}{1+R^*(1+N)} \end{pmatrix} \end{aligned}$$

It is easy to see that $NE[C^{\omega}_{T,1}]\frac{d\rho}{dN} = (1-\rho)N\frac{dE[C^{\omega}_{T,1}]}{dN}$. This implies that the second and third term in the expression above for $\bar{C}_{T,1}$ cancel out, and we find that

$$\bar{C}_{T,1} = \frac{d(1-\rho)NE[C^{\omega}_{T,1}]}{dN} = (1-\rho)E[C^{\omega}_{T,1}]$$

Economically, the reason is the following. $NE[C^{\omega}_{T,1}]\frac{d\rho}{dN}$ is the reduction in insurance costs for the pre-existing N units of insurance when the amount N is raised. $(1-\rho)N\frac{dE[C^{\omega}_{T,1}]}{dN}$ is the reduction in the payoff of the pre-existing N units of insurance that stems from a fall in average consumption $E[C^{\omega}_{T,1}]$ when the amount N is increased. The two terms cancel out because all in all, increasing N does not affect the relative costs and payoffs of the existing insurance arrangement. Lenders do not care about whether they provide a defined contingent payoff through a small number of strongly fluctuating assets or a large number of mildly fluctuating assets.

C Derivation of $\bar{C}_{T,1}$ in Model with Borrowing Constraints

As in the benchmark model, we define $\bar{C}_{T,1} = \frac{d(1-\rho)NE[C^{\omega}_{T,1}]}{dN}$ as the increase in borrowers' fixed payoff $(1-\rho)NE[C^{\omega}_{T,1}]$ from contracting more local currency debt, taking into account all general equilibrium effects. This can again be decomposed into the three parts

$$\bar{C}_{T,1} = (1-\rho)E[C^{\omega}_{T,1}] - NE[C^{\omega}_{T,1}]\frac{d\rho}{dN} + (1-\rho)N\frac{dE[C^{\omega}_{T,1}]}{dN}$$

In the following, we first express the derivative $\frac{dC^{\omega}_{T,1}}{dN}$ as an intermediate result. We then analyze the two derivatives $\frac{dE[C^{\omega}_{T,1}]}{dN}$ and $\frac{d\rho}{dN}$ and return to characterizing $\bar{C}_{T,1}$.

C.1 Derivative $dC^{\omega}_{T,1}/dN$

We can derive consumption in constrained states $C^{\text{con}}_{T,1}$ and in unconstrained states $C^{\text{unc}}_{T,1}$ from equations (31) and (32) with respect to N as

$$\frac{dC^{\text{con}}_{T,1}}{dN} = -\frac{C^{\text{con}}_{T,1} - \bar{C}_{T,1}}{1+N-\kappa\varsigma} \quad \text{and} \quad \frac{dC^{\text{unc}}_{T,1}}{dN} = -\frac{R^*\left[C^{\text{unc}}_{T,1} - \bar{C}_{T,1}\right]}{1+R^*(1+N)}$$

This can be summarized as

$$\frac{dC^{\omega}_{T,1}}{dN} = -x^{\omega}\left(C^{\omega}_{T,1} - \bar{C}_{T,1}\right) \quad \text{where} \quad x^{\omega} = \frac{dC^{\omega}_{T,1}}{dY^{\omega}_{T,1}} = \begin{cases} \frac{1}{1+N-\kappa\varsigma} & \text{if } Y^{\omega}_{T,1} < \hat{Y}_{T,1} \\ \frac{R^*}{1+R^*(1+N)} & \text{if } Y^{\omega}_{T,1} \geq \hat{Y}_{T,1} \end{cases} \tag{38}$$

Increasing the amount of insurance N raises consumption in states in which it is lower than $\bar{C}_{T,1}$ and lowers it in those where consumption is higher than $\bar{C}_{T,1}$. We use x^{ω} as the short notation for the multiplier of changes in income on consumption, as derived in proposition 4. As already discussed there, $x^{\text{con}} > x^{\text{unc}}$, i.e. the smoothing impact of more local currency debt N on consumption is stronger in constrained states than in unconstrained states.

C.2 Derivative $dE[C^{\omega}_{T,1}]/dN$

The derivative of expected consumption can then be written as[19]

$$\begin{aligned}\frac{dE[C^{\omega}_{T,1}]}{dN} &= E\left[\frac{dC^{\omega}_{T,1}}{dN}\right] = E\left[x^{\omega}\left(\bar{C}_{T,1} - C^{\omega}_{T,1}\right)\right] = E\left(x^{\omega}\right)\bar{C}_{T,1} - E\left[x^{\omega}C^{\omega}_{T,1}\right] = \\ &= E\left(x^{\omega}\right)\left(\bar{C}_{T,1} - E[C^{\omega}_{T,1}]\right) - \text{Cov}\left(x^{\omega}, C^{\omega}_{T,1}\right)\end{aligned}$$

[19]Note that we apply the Leibniz rule in the first step, but the derivatives of the boundary between constrained and unconstrained states $d\hat{Y}_{T,1}/dN$ cancel out, since at $\hat{Y}_{T,1}$ the constrained and unconstrained values of consumption are identical, i.e. consumption is a continuous function of the output shock.

Expected consumption generally falls as the amount of insurance N is increased, since insurance is costly. Algebraically, this is captured by the observation that $\bar{C}_{T,1} < E[C^{\omega}_{T,1}]$. However, as expressed by the covariance term, this direct effect is counteracted by the lower incidence of borrowing constraints, which tends to raise the level consumption: Since $x^{\text{con}} > x^{\text{unc}}$, the impact of insurance in constrained states, which is generally positive since constraints occur in low states, is stronger than the impact in unconstrained states, which is on average negative since unconstrained states are high states. This can reverse the sign of the derivative when the probability of borrowing constraints is sufficiently high.

C.3 Derivative $d\rho/dN$

Taking the total differential of equation (11), which defines the risk premium that lenders charge for a given distribution of period 1 consumption, we obtain

$$\frac{d\rho}{dN} E\left[C^{\omega}_{T,1}\right] = -\rho \frac{dE[C^{\omega}_{T,1}]}{dN} - R^{*}\text{Cov}\left(\frac{dC^{\omega}_{T,1}}{dN}, M^{\omega}_{1}\right)$$

Increasing the amount of insurance generally reduces the risk premium, as the (positive) covariance term in the expression outweighs the (generally negative) derivative term.

C.4 Solving for $\bar{C}_{T,1}$

Let us successively substitute both derivatives back into the expression for $\bar{C}_{T,1}$:

$$\begin{aligned}
\bar{C}_{T,1} &= (1-\rho)E[C^{\omega}_{T,1}] + N\left[\rho\frac{dE[C^{\omega}_{T,1}]}{dN} + R^{*}\text{Cov}\left(\frac{dC^{\omega}_{T,1}}{dN}, M^{\omega}_{1}\right)\right] + \\
&\quad +(1-\rho)N\frac{dE[C^{\omega}_{T,1}]}{dN} = \\
&= (1-\rho)E[C^{\omega}_{T,1}] + NR^{*}\text{Cov}\left(\frac{dC^{\omega}_{T,1}}{dN}, M^{\omega}_{1}\right) + N\frac{dE[C^{\omega}_{T,1}]}{dN} = \\
&= (1-\rho)E[C^{\omega}_{T,1}] + NR^{*}E\left[\frac{dC^{\omega}_{T,1}}{dN}\cdot M^{\omega}_{1}\right]
\end{aligned}$$

D Derivative $d\hat{Y}_{T,1}/dN$

Taking the derivative of $\hat{Y}_{T,1}$ with respect to N and using the definition that $\bar{C}_{T,1} = \frac{d(1-\rho)NE[C^{\omega}_{T,1}]}{dN}$, we find that

$$\begin{aligned}
\frac{d\hat{Y}_{T,1}}{dN} &= \frac{\bar{Y}_{T} - (1+\kappa\varsigma R^{*})\frac{d(1-\rho)NE[C^{\omega}_{T,1}]}{dN} - \kappa R^{*}\hat{Y}_{T,1}}{1+\kappa\left[1+R^{*}\left(1+N+\varsigma\right)\right]} = \\
&= \frac{1+\kappa\varsigma R^{*}}{1+\kappa\left[1+R^{*}\left(1+N+\varsigma\right)\right]}\cdot\left[\hat{C}_{T,1} - \bar{C}_{T,1}\right]
\end{aligned}$$

For the last step, observe that at the cutoff value $\hat{Y}_{T,1}$, the constrained and unconstrained levels of borrowing F_2^{con} and F_2^{unc} have to equal, which implies

$$F_2^{\text{con}} = \kappa(\hat{Y}_{T,1} + \varsigma\hat{C}_{T,1}) = \frac{\bar{Y}_T - \hat{C}_{T,1}}{R^*} = F_2^{\text{unc}} \quad \text{or} \quad \bar{Y}_T - \kappa R^* \hat{Y}_{T,1} = (1 + \kappa\varsigma R^*)\hat{C}_{T,1}$$